14 Examples for the design of packed columns

The relationships discussed in the previous chapters are important for the design and operation of countercurrent packed columns. Examples taken from the fields of desorption, absorption, rectification, and liquid-liquid extraction shall now be given to demonstrate how they can be applied in plant engineering. It is presumed that the reader is acquainted with the principles underlying thermal separation techniques.

Fig. 14.1 shows a flow chart for a rectification column with a concentration $x$ of low boilers in the feed $F$, overhead product $D$, and bottoms $B$. These fractions are presented qualitatively in the $y/x$ diagram. The example concerned is the separation of a binary mixture; the equilibrium curve $y_e = f(x)$ and the operating lines $y = f(x)$, viz. $BI$ in the stripping zone and $ID$ in the enrichment zone, are given.

![Diagram of Rectification Column]

Fig. 14.1. Qualitative determination of transfer units in rectification

$h_F$ = feed enthalpy
$h_F'$ = feed enthalpy at boiling temperature
$\Delta h_v$ = vaporization enthalpy at feed section

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Fig. 14.2 illustrates the principles of absorption and desorption. The factors given are the gas volumetric flow rate $V$, the concentration of transfer components $y$ in the gas, the liquid volumetric flow rate $L$, and the concentration of transfer components $x$ in the liquid. The subscript $u$ indicates the bottom of the column; and the subscript $o$, the head of the column.

The gas load $Y$ of transfer component is related to the mole fraction $y$ by

$$Y = \frac{y}{1 - y} \quad (14-1)$$

Likewise, the liquid load $X$ of transfer component is related to the mole fraction $x$ by

$$X = \frac{x}{1 - x} \quad (14-2)$$

The number of transfer units $NTU_{OV}$ required for rectification between zones with molar fractions $y_a$ and $y_b$ is given by

$$NTU_{OV} = \frac{y_u - y_o}{\Delta y_a - \Delta y_b} \ln \frac{\Delta y_a}{\Delta y_b} \quad (14-3)$$

The terms $\Delta y_a$ and $\Delta y_b$ in this equation represent the difference between the equilibrium mole fraction $y_{e,a}$ or $y_{e,b}$ and the vapour mole fraction $y_a$ or $y_b$, i.e.

$$\Delta y_a = y_{e,a} - y_a \quad (14-4)$$

$$\Delta y_b = y_{e,b} - y_b \quad (14-5)$$

The size of the zones should be selected so that the slope $m_{yx}$ of the equilibrium curve is practically the same in each.

In contrast, the slopes of the equilibrium curves in absorption and desorption processes can be regarded as constant over the entire range of concentrations between the head and the foot of the column, particularly at low phase loads. In this case, if the gas load is adopted as a measure for the concentration, the number of transfer units in absorption processes will be given by

$$NTU_{OV} = \frac{Y_u - Y_o}{\Delta Y_u - \Delta Y_o} \ln \frac{\Delta Y_u}{\Delta Y_o} \quad (14-6)$$

where $Y_u$ and $Y_o$ are the gas loads at the foot and head of the column. The terms $\Delta Y_u$ and $\Delta Y_o$ are then defined by

$$\Delta Y_u = Y_u - m_{yx} X_u \quad (14-7)$$

$$\Delta Y_o = Y_o - m_{yx} X_o \quad (14-8)$$

The corresponding equations for the liquid load are

$$NTU_{OL} = \frac{X_u - X_o}{\Delta X_u - \Delta X_o} \ln \frac{\Delta X_u}{\Delta X_o} \quad (14-9)$$
$$\Delta X_u = \frac{1}{m_{yx}} Y_u - X_u$$  \hspace{1cm} (14-10)

$$\Delta X_o = \frac{1}{m_{yx}} Y_o - X_o$$  \hspace{1cm} (14-11)

Likewise, the number of transfer units required in desorption processes will be given by Eqns (14-12) to (14-14) if the gas load is adopted as the measure for the concentration:

$$NTU_{ov} = \frac{Y_o - Y_u}{\Delta Y_o - \Delta Y_u} \ln \frac{\Delta Y_o}{\Delta Y_u}$$  \hspace{1cm} (14-12)

---

Absorption

$$\text{Liquid}$$

$$\text{Gas}$$

---

$$L/V$$ Molar liquid-to-gas ratio or slope of the operating line

$$X,Y$$ Load fraction of transfer component in liquid/gas

$$L,U$$ Carrier stream of liquid/gas phase

$$X_u = X_o - \frac{1}{L/V} (Y_u - Y_o)$$

$$\frac{L}{V} = \frac{Y_o - Y_u}{X_o - X_u}$$

$$Y_o = Y_u + \frac{L}{V} (X_o - X_u)$$

Fig. 14.2. Qualitative determination of transfer units in absorption and desorption
14 Examples for the design of packed columns

\[ \Delta Y_o = m_{YX} X_o - Y_o \]  \hspace{1cm} (14-13)

\[ \Delta Y_u = m_{YX} X_u - Y_u \]  \hspace{1cm} (14-14)

The corresponding equations for the liquid phase are

\[ NTU_{OL} = \frac{X_o - X_u}{\Delta X_o - \Delta X_u} \ln \frac{\Delta X_o}{\Delta X_u} \]  \hspace{1cm} (14-15)

\[ \Delta X_o = X_o - \frac{1}{m_{YX}} Y_o \]  \hspace{1cm} (14-16)

\[ \Delta X_u = X_u - \frac{1}{m_{YX}} Y_u \]  \hspace{1cm} (14-17)

In a rectification process, as illustrated in Fig. 14.1, the reflux ratio is given by

\[ r > r_{min} = \frac{x_D - x_f^*}{y_f^* - x_f^*} \]  \hspace{1cm} (14-18)

The molar flow rate at the head of the column would then be

\[ V = \hat{F}_{\text{L}} \frac{x_F - x_B}{x_D - x_B} (r + 1) \]  \hspace{1cm} (14-19)

In an absorption process, as illustrated in Fig. 14.2 (left), the total liquid-vapour ratio must be higher than the ratio of the carrier liquid, i.e. solvent, to the vapour, i.e.

\[ \frac{L}{V} > \frac{L_m}{V} \]  \hspace{1cm} (14-20)

The flow rate of solvent in this case is given by

\[ L_m = V \frac{Y_u - X_o}{X_{u,m} - X_o} \]  \hspace{1cm} (14-21)

In desorption, as illustrated in Fig. 14.2 (right), the total vapour/liquid ratio must be higher than the ratio of the stripping gas to the liquid, i.e.

\[ \frac{V}{L} > \frac{V_m}{L} \]  \hspace{1cm} (14-22)

The flow rate of stripping gas in this case is given by

\[ V_m = L \frac{X_o - X_u}{Y_{o,m} - Y_u} \]  \hspace{1cm} (14-23)
In absorption and desorption, the total molar flow rate of vapour, consisting of the carrier gas and the transfer components, is given by

\[ V = V(1 + Y) \text{ where } V = V_u(1 - y_u) \]  \hspace{1cm} (14-24)

and the total molar flow rate of liquid, consisting of the carrier liquid and the transfer components, by

\[ L = L(1 + X) \text{ where } L = L_o(1 - x_o) \]  \hspace{1cm} (14-25)

The column diameter \( d_s \) can be expressed in terms of the free cross-sectional area \( A_s \), i.e.

\[ d_s = \sqrt{\frac{4}{\pi} A_s} \]  \hspace{1cm} (14-26)

If \( u_V \) is the superficial gas velocity, \( A_s \) can be defined by

\[ A_s = \frac{\dot{V}}{u_V} \frac{u_V}{\dot{Q}_V} \]  \hspace{1cm} (14-27)

Thus the column diameter can be calculated from the operating parameters \( \dot{V} \) [kmol/h], \( u_V \) [m/s], \( T \) [K], and \( p \) [bar], i.e.

\[ d_s = 5.4 \cdot 10^{-3} \sqrt{\frac{\dot{V}}{p} \frac{T}{u_V}} \]  \hspace{1cm} (14-28)

Fig. 14.3 illustrates liquid-liquid extraction in which the specific gravity of the continuous phase (Subscript C) is greater than that of the dispersed phase (Subscript D). In this case, the total number of transfer units \( NTU_{OC} \) derived from the \( HTU-NTU \) model is given by

\[ NTU_{OC} = \frac{C_{C,o} - C_{C,u}}{c_{D,o} - c_{D,u}} \ln \frac{\Delta c_o}{\Delta c_u} \]  \hspace{1cm} (14-29)

where \( \Delta c_o \) is the driving concentration difference at the head of the column, as defined by Eqn (14-30), and \( \Delta c_u \) is the driving concentration difference at the foot of the column, as defined by Eqn (14-31), i.e.

\[ \Delta c_o = c_{C,o} - \frac{1}{m} c_{D,o} \]  \hspace{1cm} (14-30)

\[ \Delta c_u = c_{C,u} - \frac{1}{m} c_{D,u} \]  \hspace{1cm} (14-31)

Another case to consider is that in which mass transfer is in the \( C \rightarrow D \) direction and the desired separation efficiency is given by

\[ \eta_E = 1 - \frac{c_{C,u}}{c_{C,o}} \]  \hspace{1cm} (14-32)
Rearranging this equation gives rise to the following:

\[ c_{C,u} = (1 - \eta_C) c_{C,o} \]  

(14-33)

from which it can be derived that the concentration of transfer components in the continuous phase decreases from \( c_{C,o} \) at the head of the column to \( c_{C,u} \) at the foot.

If the phase ratio in the extractor is \( u_C/u_D \), the concentration of transfer components in the dispersed phase increases from a value of \( c_{D,u} \) at the foot of the column to the following value of \( c_{D,o} \) at the head of the column:

\[ c_{D,o} = \frac{u_C}{u_D} (c_{C,o} - c_{C,u}) + c_{D,u} \]  

(14-34)

Eqn (13-22) allows the effective height \( H \) to be determined from NTU \( O_C \) and HTU \( O_C \); and, if the volume flow rate \( \dot{V}_C \) of the continuous phase is given, the column diameter \( d_t \) can be obtained from the load \( u_C < 0.7 u_{C,fl} \) from the following equation:

\[ d_t = \sqrt{\frac{4 \dot{V}_C}{\pi u_C}} \]  

(14-35)

14.1 Determination of the diameter of an off-gas absorption column with various types of packing

Polluted air flowing at a rate of \( 10^5 \) m\(^3\)/h STP is to be scrubbed with water in a packed column. The liquid-to-gas ratio must be varied between \( L/V = 1 \) and \( L/V = 10 \), depending on the degree of contamination. The maximum permissible load is that at the loading point.
14.1 Determination of the diameter of an off-gas absorption column with various types of packing

Under these conditions, what diameter would the column have to be if 50-mm packing of the following types had to be installed?

- Plastics packing: Intalox saddles, Bialecki rings, Pall rings, and Norpac rings
- Metal packing: Pall rings, Hiflow rings, Bialecki rings, and VSP rings

**Solution**

**Properties of the system**

Density and dynamic viscosity of the gas

\[ \rho_V = 1.19 \text{ kg/m}^3 \]
\[ \eta_V = 18 \cdot 10^{-6} \text{ kg/ms} \]

Density and dynamic viscosity of liquid

\[ \rho_L = 998 \text{ kg/m}^3 \]
\[ \eta_L = 1 \cdot 10^{-3} \text{ kg/ms} \]

The values of \( C_s \) required to calculate the resistance coefficients \( \xi_s \) at the loading point from Eqn (4-29) are listed in Table 4.1.

![Graph showing column diameter as a function of liquid-vapour ratio for various packing sizes and types](image)

**Fig. 14.4.** Column diameter as a function of the liquid-vapour ratio for the data given in Section 14.1
The values of $\xi$, calculated from Eqn (4-29) for all the packings, together with the associated values for $a$ and $e$, which are also listed in Table 4.1, allow the corresponding vapour flow rates at the loading point $u_V = u_{VS}$ to be determined by iteration as functions of the liquid/vapour ratio $L/V$ from Eqn (11-82).

The evaluated results are plotted in Fig. 14.4. It can be seen that the column diameter $d_s$ required for a ratio $L/V = 10$ is about 25% greater than that for $L/V = 1$. The diagram also shows that the differences in diameter associated with plastics packing are greater than those for metal packing.

### 14.2 Design of a packed column for the rectification of an isobutane/n-butane mixture

An isobutane/n-butane mixture flowing at a rate of 500 kmol/h has to be separated in a rectification column. It consists of 60% of the low-boiling component and 40% of the high-boiling and enters the column at the boiling point. The overhead product has to consist of 99.9% mol of isobutane, and the bottoms may contain only 0.1% mol of this component. The packing is of the Montz Bl-200 sheet-metal type, and the column operates at an overhead pressure of 3.6 bar and a reflux ratio of 15 under the conditions at the loading point.

**Solution**

**Average properties of the system**

\[
\begin{align*}
\mu_V &= 58.124 \text{ kg/kmol} \\
\varrho_V &= 8.398 \text{ kg/m}^3 \\
\eta_V &= 7.76 \cdot 10^{-6} \text{ kg/ms} \\
D_V &= 1.17 \cdot 10^{-6} \text{ m}^2/\text{s} \\
\alpha_m &= 1.35
\end{align*}
\]

\[
\begin{align*}
\mu_L &= 58.124 \text{ kg/kmol} \\
\varrho_L &= 561.3 \text{ kg/m}^3 \\
\eta_L &= 0.13078 \cdot 10^{-3} \text{ kg/ms} \\
D_L &= 8.44 \cdot 10^{-9} \text{ m}^2/\text{s} \\
\alpha_L &= 8.88 \cdot 10^{-3} \text{ kg/s}^2
\end{align*}
\]

**Characteristic data for the packing derived from Tables 4.2, 4.4, and 5.2**

\[
\begin{align*}
a &= 200 \text{ m}^2/\text{m}^3 & C_s &= 3.116 & C_V &= 0.390 \\
e &= 0.979 \text{ m}^3/\text{m}^3 & C_L &= 0.971 & C_P &= 0.355
\end{align*}
\]

**Product streams designated as in Fig. 14.1**

\[
\begin{align*}
\dot{D} &= 500 \text{ kmol/h} \cdot \frac{0.6-0.001}{0.999-0.001} = 300 \text{ kmol/h} \\
L' &= 300.1 \text{ kmol/h} \cdot 15 = 4501.5 \text{ kmol/h} \\
&= 4501.5 \text{ kmol/h} \cdot 58.124 \text{ kg/kmol} = 261645 \text{ kg/h} \\
V' &= 300.1 \text{ kmol/h} (15 + 1) = 4801.6 \text{ kmol/h} \\
&= 4801.6 \text{ kmol/h} \cdot 58.124 \text{ kg/kmol} = 279088 \text{ kg/h}
\end{align*}
\]
14.2 Design of a packed column for the rectification of an isobutane/n-butane mixture

Resistance coefficient at the loading point according to Eqn (4-29)

\[
\hat{\xi}_S = \left[ \frac{9.806}{3.116^2} \left( \frac{261645}{279088} \right)^{1/2} \left( \frac{8.398}{561.3} \right)^{0.514} \left( \frac{0.131 \cdot 10^{-3} \cdot 279088}{7.76 \cdot 10^{-6}} \right)^{0.514} \left( \frac{0.131 \cdot 10^{-3}}{7.76 \cdot 10^{-6}} \right)^{0.514} \right]^{-0.652} = 0.514
\]

Relationship at the loading point according to Eqn (4-28)

\[
u_{VS} = \left( \frac{9.806}{0.514} \right)^{1/2} \left( \frac{0.979}{200^{1/6} - 200^{1/2}} \left( \frac{12}{9.806} \cdot \frac{0.131 \cdot 10^{-3} \cdot u_L}{561.3} \right)^{1/3} \right) \cdot \left( \frac{12}{9.806} \cdot \frac{0.131 \cdot 10^{-3} \cdot u_L}{561.3} \right)^{1/6} \left( \frac{561.3}{8.398} \right)^{1/2}
\]

Relationship at the loading point according to Eqn (11-82) for \( u_L \) and \( u_V = u_{VS} \)

\[
u_L = \frac{261645}{279088} \cdot \frac{8.398}{561.3} \cdot \frac{u_{VS}}{561.3}
\]

Vapour load at the loading point determined by solving Eqns (4-28) and (11-82) by iteration

\[
u_{VS} = 0.489 \text{ m/s}
\]

Cross-sectional area of column according to Eqn (14-27)

\[
A_s = \frac{279088}{3600} \cdot \frac{0.489 \cdot 8.398}{0.489 \cdot 8.398} = 18.88 \text{ m}^2
\]

Column diameter according to Eqn (14-26)

\[
d_s = \sqrt{\frac{4}{\pi} \cdot 18.88} = 4.9 \text{ m}
\]

Column diameter selected for planning: \( d_s = 5 \text{ m} \).

Cross-sectional area of the planned column

\[
A_s = \frac{\pi}{4} \cdot 5^2 = 19.635 \text{ m}^2
\]

Vapour and liquid loads in the planned column

\[
u_V = \frac{279088}{3600} \cdot \frac{8.398}{8.398 \cdot 19.635} = 0.47 \text{ m/s}
\]

\[
u_L = \frac{261645}{561.3 \cdot 19.635} = 6.59 \cdot 10^{-3} \text{ m}^3/\text{m}^2/\text{s}
\]
Examples for the design of packed columns

Theoretical liquid holdup according to Eqn (4-27)

\[ h_L = \left( 12 \cdot \frac{0.131 \cdot 10^{-3} \cdot 6.59 \cdot 10^{-3} \cdot 200^2}{9.806 \cdot 561.3} \right)^{1/3} \]

\[ = 0.0422 \text{ m}^3/\text{m}^2 \]

Particle diameter according to Eqn (4-66)

\[ d_p = 6 \cdot \frac{1 - 0.979}{200} = 0.63 \cdot 10^{-3} \text{ m} \]

Factor to allow for end effects {Eqn (4-57)}

\[ f_s = \left( 1 + \frac{4}{200 \cdot 5} \right)^{-1} = 0.996 \]

Reynolds number for the stream of vapour {Eqn (4-65)}

\[ Re_v = \frac{0.47 \cdot 0.63 \cdot 10^{-3} \cdot 8.398}{(1 - 0.979) \cdot 7.76 \cdot 10^{-6} \cdot 0.996} = 15200 \]

Reynolds number for the stream of liquid {Eqn (4-69)}

\[ Re_L = \frac{6.59 \cdot 10^{-3} \cdot 561.3}{200 \cdot 0.131 \cdot 10^{-3}} = 141.42 \]

Wetting factor for the packing {Eqn (4-68)}

\[ W = \exp \left( \frac{141.42}{200} \right) = 2.028 \]

Resistance coefficient for the stream of vapour {Eqn (4-67)}

\[ \xi_L = 0.355 \cdot 2.028 \left( \frac{64}{15200} + \frac{1.8}{15200^{0.08}} \right) \left( \frac{0.979 - 0.0422}{0.979} \right)^{1.5} = 0.564 \]

Pressure drop in the vapour stream {Eqn (4-58)}

\[ \frac{\Delta p}{H} = 0.564 \cdot \frac{200}{(0.979 - 0.0422)^3} \cdot \frac{8.398 \cdot 0.47^2}{2} \cdot \frac{1}{0.996} = 128.3 \text{ Pa/m} \]

Hydraulic diameter {Eqn (5-15)}

\[ d_h = 4 \cdot \frac{0.979}{200} = 0.01958 \text{ m} \]
14.2 Design of a packed column for the rectification of an isobutane/n-butane mixture

Area of phase contact \{Eqn (5-16)\}

\[
\frac{a_{ph}}{a} = 3 \cdot 0.979^{0.5} \left( \frac{6.59 \cdot 10^{-0.3} \cdot 561.3}{200 \cdot 0.13078 \cdot 10^{-3}} \right)^{-0.2} \cdot \left( \frac{(6.59 \cdot 10^{-3})^2 \cdot 561.3}{0.00888 \cdot 200} \right)^{0.75} \left( \frac{6.59 \cdot 10^{-3} \cdot 200}{9.806} \right)^{-0.45} = 1.045
\]

Height of a transfer unit in the liquid phase \{Eqn (5-5)\}

\[
HTU_L = \frac{1}{0.971} \left( \frac{0.13078 \cdot 10^{-3}}{561.3 \cdot 9.806} \right)^{1/6} \left( \frac{0.01958}{8.44 \cdot 10^{-9}} \right)^{1/2} \left( \frac{6.59 \cdot 10^{-3}}{200} \right)^{2/3} \frac{1}{1.045} = 0.083 \text{ m}
\]

Height of a transfer unit in the vapour phase \{Eqn (5-14)\}

\[
HTU_V = \frac{1}{0.390} (0.979 - 0.0422)^{1/2} \cdot \frac{0.01958^{1/2}}{200^{3/2}} \cdot \frac{0.47 \cdot 8.398}{1.17 \cdot 10^{-6}} \cdot \left( \frac{200 \cdot 7.76 \cdot 10^{-6}}{0.47 \cdot 8.398} \right)^{3/4} \left( \frac{1.17 \cdot 10^{-6} \cdot 8.398}{7.76 \cdot 10^{-6}} \right)^{1/3} \frac{1}{1.045} = 0.143 \text{ m}
\]

Determination of the concentration zones (Table 14.1)

Calculation for the height of the uppermost zone

\[x_a = x_D = 0.999 \text{ kmol/kmol to } x_b = 0.8 \text{ kmol/kmol}\]

Characteristic concentration in the uppermost zone

Phase equilibrium concentration corresponding to \(y_a = x_a = 0.999 \text{ kmol/kmol} \{Eqn (1-38)\}\)

\[
y_{e,a} = \frac{1.35 \cdot 0.999}{1 + (1.35 - 1) \cdot 0.999} = 0.9993 \text{ kmol/kmol}
\]

Vapour concentration corresponding to \(x_b = 0.8 \text{ kmol/kmol} \) on the operating line for the enrichment zone \{Eqn (1-13)\}

\[
y_b = \frac{15}{15 + 1} \cdot 0.8 + \frac{0.999}{15 + 1} = 0.8124 \text{ kmol/kmol}
\]

Phase equilibrium concentration

\[
y_{e,b} = \frac{1.35 \cdot 0.8}{1 + (1.35 - 1) \cdot 0.8} = 0.8438 \text{ kmol/kmol}
\]
Differences in concentration \{Eqns (14-4) and (14-5)\}

\[
\Delta y_a = 0.9993 - 0.999 = 0.0003 \text{ kmol/kmol}
\]
\[
\Delta y_b = 0.8438 - 0.8124 = 0.0314 \text{ kmol/kmol}
\]

Number of vapour-side transfer units \{Eqn (14-3)\}

\[
NTU_{ov} = \frac{0.999 - 0.8124}{0.0003 - 0.0314} \cdot \ln \frac{0.0003}{0.0314} = 28.8
\]

Mean slope of equilibrium curve \{Eqn (1-66)\}

\[
m_{yx} = \frac{1.3}{[1 + (1.3 - 1) 0.867]^2} = 0.8184
\]

Stripping factor \{Eqn 1-30\})

\[
\lambda = 0.8184 \cdot \frac{4801.6 \text{ kmol/h}}{4501.5 \text{ kmol/h}} = 0.8335
\]

Number of theoretical stages \{Eqn (1-27)\}

\[
n_t = 28.8 \cdot \frac{0.8335 - 1}{\ln 0.8335} = 26.33
\]

Height of an overall transfer unit \{Eqn (1-62)\}

\[
HTU_{ov} = 0.143 + 0.8335 \cdot 0.083 = 0.212 \text{ m}
\]

Height of uppermost concentration zone \{Eqn (1-64)\} if no allowance is made for end effects

\[
H = 0.212 \cdot 28.8 = 6.106 \text{ m}
\]

The results thus obtained for all the concentration zones are listed in Table 14.2.

Total theoretical height if no allowance is made for end and distribution effects

\[
\Sigma H = 15 \text{ m.}
\]

14.3 Scaling up measurements performed in pilot plants

In mass transfer studies in a column with a new type of packing, a total of \(N_{Pa} = 5.6\) transfer units was measured for a height of \(H_{Pa} = 1.5 \text{ m}\). Hence the number of transfer units per unit height of bed was \((N/H)_{Pa} = 3.733 \text{ m}^{-1}\). In a control experiment with a bed height of \(H_{Pb} = 2 \text{ m}\), the number of transfer units measured was \(N_{Pb} = 6.8\). What would the error
be if the efficiency determined for a height $H_{Pa} = 1.5$ m were to be taken as a basis in calculating the height $H_T$ of an industrial-scale column required to realize $N_T = 20$ transfer units on the assumption that the inlet distribution remains unchanged? For the purpose of the calculation, assume that the height of the inlet zone $H_i$ is less than $H_{Pa} = 1.5$ m.

**Solution**

If no allowance is made for end effects, the height of the column would be

$$H_{T(P)} = \frac{N_T}{(N/H)_{Pa}} = \frac{20}{3.733} = 5.36 \text{ m}$$

If it is assumed that the separation efficiency remains constant in the $H > H_i$ zone, the following applies for $H < H_p$

$$\left( \frac{dN}{dH} \right)_{H > H_i} = \frac{N_{Pb} - N_{Pa}}{H_{Pb} - H_{Pa}} = \frac{6.8 - 5.6}{2 - 1.5} = 2.4 \text{ m}^{-1}$$

According to the model presented in Fig. 8.4, the end effects $(N_d + \Delta N_i)$ can be determined by rearranging Eqn (6-8) for the case of $H = H_{Pa}$, i.e.

$$N_d + \Delta N_i = N_{Pa} - H_{Pa} \left( \frac{dN}{dH} \right)_{H > H_i}$$

$$= 5.6 - 1.5 \cdot 2.4 = 2$$

The relative difference in height required to compensate end effects can then be obtained from Eqn (8-15), i.e.

$$\frac{\Delta H}{H_T} = 1 - \frac{1}{1 - \frac{2}{20} \cdot \frac{1}{1 + \frac{2}{1.5} \cdot \frac{1}{2.4}}} = 0.285$$

According to Eqn (6-35), this corresponds to an efficiency ratio of

$$\eta_T = 1 - \frac{\Delta H}{H_T} = 1 - 0.285 = 0.715$$

The actual column height required can then be calculated from Eqns (6-35) and (6-39), i.e.

$$H_T = \frac{1}{\eta_T} H_{T(P)} = \frac{1}{0.715} \cdot 5.357 = 7.49 \text{ m}$$
The same result can be obtained from Eqn (6-28). Thus

\[
H_T = \frac{1}{1 - \eta_T} \left( \frac{N}{N_d + \Delta N} \right)_{p,a}
\]

\[
= \frac{1}{1.5 - 0.715 \cdot 3.733} = 7.49 \text{ m}
\]

Hence, the absolute value for the additional height required is

\[
\Delta H = \frac{\Delta H}{H_T} H_T = 0.285 \cdot 7.49 = 2.13 \text{ m}.
\]

This figure agrees with that for the difference between the values calculated for \(H_T\) and \(H_{T(P)}\), i.e.

\[
\Delta H = H_T - H_{T(P)} = 7.49 - 5.36 = 2.13 \text{ m}.
\]

Hence, if the column design were to be based on the measured value for \((N/H)_{p,a} = 3.733 \text{ m}^{-1}\), the height of the bed would be too short by an amount

\[
\frac{\Delta H}{H_T} 100 = 28.5\%.
\]

### 14.4 Design of an absorber for removing acetone from process air

The spent air from a production plant contains 1% mol of acetone and flows at a rate of 2000 m³/h at 27°C. The acetone has to be removed to a final concentration of 100 mg/m³ by absorption in water in a column packed with 50-mm metal Pall rings. The main dimensions of the column, which is operated at the loading point, have to be determined for the case in which the water consumption is 1.4 times higher than the minimum.

**Solution**

Average properties of the gas phase

\[
\mu_V = 28.96 \text{ kg/kmol} \\
\nu_V = 1.162 \text{ kg/m}^3 \\
\eta_V = 18.13 \cdot 10^{-6} \text{ kg/ms} \\
D_V = 10.8 \cdot 10^{-6} \text{ m}^2/\text{s}
\]

Molar mass of acetone

\[
\mu_A = 58.08 \text{ kg/kmol}
\]
Average properties of the liquid phase

\[ \mu_L = 18.02 \text{ kg/kmol} \]
\[ \varrho_L = 997 \text{ kg/m}^3 \]
\[ \eta_L = 0.857 \cdot 10^{-3} \text{ kg/ms} \]
\[ D_L = 1.18 \cdot 10^{-9} \text{ m}^2/\text{s} \]
\[ \alpha_L = 72 \cdot 10^{-3} \text{ kg/s}^2 \]

Slope of equilibrium curve for the acetone-air/water absorption system

\[ m_{YX} = 2.314 \]

Packing characteristics (Tables 4.1, 4.3, 4.6 and 5.1)

\[ a = 110 \text{ m}^2/\text{m}^3 \]
\[ e = 0.952 \text{ m}^3/\text{m}^3 \]
\[ C_s = 2.725 \]
\[ C_p = 0.763 \]
\[ C_L = 1.192 \]
\[ C_h = 0.784 \]
\[ C_Y = 0.410 \]

Product streams (Fig. 14.2, left)

\[ \dot{V}_u = 2000 \frac{\text{m}^3}{h} \cdot \frac{1.162 \text{ kg/m}^3}{28.96 \text{ kg/kmol}} = 80.25 \text{ kmol/h} \]

\[ y_u = 0.01 \text{ kmol/kmol} \]
\[ Y_u = \frac{0.01}{1-0.01} = 0.01 \text{ kmol/kmol} \]
\[ V = 80.25 (1-0.01) = 79.45 \text{ kmol/h} \]

\[ y_o = 100 \cdot 10^{-6} \frac{\text{kg}}{\text{m}^3} \cdot \frac{28.96 \text{ kg/kmol}}{58.08 \text{ kg/kmol} \cdot 1.162 \text{ kg/m}^3} \]
\[ = 4.291 \cdot 10^{-5} \text{ kmol/kmol} \]
\[ Y_o = \frac{4.291 \cdot 10^{-5}}{1 - 4.291 \cdot 10^{-5}} = 4.291 \cdot 10^{-5} \text{ kmol/kmol} \]
\[ X_{u,m} = \frac{0.01}{2.314} = 4.322 \cdot 10^{-3} \text{ kmol/kmol} \quad \{\text{Eqn (1-67)}\} \]
\[ L_m = 79.45 \cdot 0.01 - 4.291 \cdot 10^{-5} \frac{4.322 \cdot 10^{-3}}{} = 183.04 \text{ kmol/h} \]
\[ L = 1.4 \cdot 183.04 = 256.26 \text{ kmol/h} = L_o \]
\[ X_u = 0 + \frac{79.45}{256.26} (0.01 - 4.291 \cdot 10^{-5}) = 3.087 \cdot 10^{-3} \text{ kmol/kmol} \]
Examples for the design of packed columns

\[ L_o = 256.26 \text{ kmol/h} \cdot 18.02 \text{ kg/kmol} = 4617.7 \text{ kg/h} \]
\[ V_o = 79.45 \left( 1 + 4.291 \cdot 10^{-5} \right) = 79.45 \text{ kmol/h} \]
\[ = 79.45 \text{ kmol/h} \cdot 28.96 \text{ kg/kmol} = 2301 \text{ kg/h} \]

Resistance coefficient \{Eqn (4-29)\}

\[ \xi_s = \frac{9.806}{2.725^2} \left[ \frac{4617.7}{2301} \left( \frac{1.162}{997} \right)^{1/2} \left( \frac{0.857 \cdot 10^{-3}}{18.13 \cdot 10^{-6}} \right)^{0.4} \right]^{-0.652} = 0.629 \]

Vapour velocity at the loading point \{Eqn (4-28)\}

\[ u_{vs} = \left( \frac{9.806}{0.629} \right)^{1/2} \left[ \frac{0.952}{110^{1/6}} - 110^{1/2} \left( \frac{12}{9.806} \cdot \frac{0.857 \cdot 10^{-3} \cdot u_L}{997} \right)^{1/3} \right] \cdot \left( \frac{12}{9.806} \cdot \frac{0.857 \cdot 10^{-3} \cdot u_L}{997} \right)^{1/6} \left( \frac{997}{1.162} \right)^{1/2} \]

Liquid velocity at the loading point \{Eqn (11-82)\} for \( u_L \) and \( u_V = u_{vs} \)

\[ u_L = \frac{4617.7}{2301} \cdot \frac{1.162}{997} u_{vs} \]

Gas velocity at the loading point determined by solving Eqns (4-28) and (11-82) by iteration

\[ u_{vs} = 1.988 \text{ m/s} \]

Cross-sectional area of column \{Eqn (14-26)\}

\[ A_s = \frac{2300.9}{3600} \cdot \frac{1.162 \cdot 1.988}{1.162 \cdot 1.988} = 0.277 \text{ m}^2 \]

Column diameter \{Eqn (14-27)\}

\[ d_s = \sqrt{\frac{4}{\pi}} \cdot 0.277 = 0.594 \text{ m} \]

Column diameter selected for planning: \( d_s = 0.6 \text{ m} \)

Cross-sectional area of planned column

\[ A_s = \frac{\pi}{4} \cdot 0.6^2 = 0.2827 \text{ m}^2 \]
Vapour and liquid loads in the planned column

\[ u_v = \frac{2300.9}{1.162 \cdot 0.2827} = 1.945 \text{ m/s} \]

\[ u_L = \frac{4617.7}{997 \cdot 0.2827} = 4.55 \times 10^{-3} \text{ m}^3/\text{m}^2\text{s} \]

Theoretical liquid holdup \( \text{Eqn (4-27)} \)

\[ h_L = \left(12 \cdot \frac{0.857 \cdot 10^{-3} \cdot 4.55 \cdot 10^{-3} \cdot 110^2}{9.806 \cdot 997} \right)^{1/3} = 0.0387 \text{ m}^3/\text{m}^3 \]

Particle diameter \( \text{Eqn (4-66)} \)

\[ d_p = 6 \cdot \frac{1 - 0.952}{110} = 2.618 \times 10^{-3} \text{ m} \]

Wall factor \( \text{Eqn (4-57)} \)

\[ f_s = \left(1 + \frac{4}{110 \cdot 0.6}\right)^{-1} = 0.9429 \]

Reynolds number for gas flow \( \text{Eqn (4-65)} \)

\[ Re_v = \frac{1.945 \cdot 2.618 \cdot 10^{-3} \cdot 1.162}{(1 - 0.952) \cdot 18.13 \cdot 10^{-6} \cdot 0.9429} = 6411 \]

Reynolds number for liquid flow \( \text{Eqn (4-69)} \)

\[ Re_L = \frac{4.55 \cdot 10^{-3} \cdot 997}{110.0 \cdot 876 \cdot 10^{-3}} = 48.12 \]

Wetting factor for the packing \( \text{Eqn (4-68)} \)

\[ W = \exp \left(\frac{48.12}{200}\right) = 1.272 \]

Resistance factor in gas flow \( \text{Eqn (4-67)} \)

\[ \xi_L = 0.763 \cdot 1.272 \left(\frac{64}{6411} + \frac{1.8}{6411^{0.08}}\right) \left(\frac{0.952 - 0.0387}{0.952}\right)^{1.5} = 0.823 \]

Pressure drop per unit height in gas flow \( \text{Eqn (4-58)} \)

\[ \frac{\Delta p}{H} = 0.823 \cdot \frac{110}{(0.952 - 0.0387)^3} \cdot \frac{1.162 \cdot 1.945^3}{2} \cdot \frac{1}{0.9429} = 277.2 \text{ Pa/m} \]
Hydraulic diameter \( \{ \text{Eqn (5-15)} \} \)

\[ d_h = 4 \cdot \frac{0.952}{110} = 0.0346 \text{ m} \]

Phase contact area \( \{ \text{Eqn (5-16)} \} \)

\[ \frac{a_{ph}}{a} = 3 \cdot 0.952^{0.5} \left( \frac{4.55 \cdot 10^{-3} \cdot 0.857}{110} \right)^{-0.2}\cdot \left( \frac{4.55 \cdot 10^{-3} \cdot 9.806}{0.072 \cdot 110} \right)^{0.75} = 0.672 \]

Height of transfer unit in the liquid phase \( \{ \text{Eqn (5-5)} \} \)

\[ HTU_L = \frac{1}{1.192} \left( \frac{0.857 \cdot 10^{-3} \cdot 997 \cdot 9.806}{1.18 \cdot 10^{-9}} \right)^{1/6} \left( \frac{0.0346}{1.18 \cdot 10^{-9}} \right)^{1/2} \left( \frac{4.55 \cdot 10^{-3} \cdot 110}{9.806} \right)^{2/3} \left( \frac{1}{0.672} \right) = 0.539 \text{ m} \]

Height of a transfer unit in the gas phase \( \{ \text{Eqn (5-14)} \} \)

\[ HTU_V = \frac{1}{0.410} \left( 0.952 - 0.0387 \right)^{1/2} \left( \frac{0.0346}{110^{3/2}} \frac{1.945}{10.8 \cdot 10^{-5}} \right) \cdot \left( \frac{1.945 \cdot 1.162}{18.13 \cdot 10^{-6}} \right)^{3/4} \left( \frac{10.8 \cdot 10^{-6} \cdot 1.162}{18.13 \cdot 10^{-6}} \right)^{1/3} = 0.456 \text{ m} \]

Differences in gas loads \( \{ \text{Eqns (14-7) and (14-8)} \} \)

\[ \Delta Y_w = 0.01 - 2.314 \cdot 3.807 \cdot 10^{-3} = 2.864 \cdot 10^{-3} \text{ kmol/kmol} \]

\[ \Delta Y_o = 4.291 \cdot 10^{-5} - 2.314 \cdot 0 = 4.291 \cdot 10^{-5} \text{ kmol/kmol} \]

Overall number of gas-side transfer units \( \{ \text{Eqn (14-6)} \} \)

\[ NTU_{ov} = \frac{0.01 - 4.291 \cdot 10^{-5}}{2.864 \cdot 10^{-3} - 4.291 \cdot 10^{-5}} \cdot \ln \frac{2.864 \cdot 10^{-3}}{4.291 \cdot 10^{-5}} = 14.826 \]

Stripping factor \( \{ \text{Eqn (1-68)} \} \)

\[ \lambda = 2.314 \cdot \frac{79.45 \text{ kmol/h}}{256.26 \text{ kmol/h}} = 0.7174 \]
Overall height of a transfer unit (Eqn (1-62))

\[ HTU_{OV} = 0.456 + 0.7174 \cdot 0.539 = 0.843 \text{ m} \]

Height of bed of packing without allowance for end and distribution effects

\[ H = 0.843 \cdot 14.826 = 12.49 \text{ m} \]

### 14.5 Design of a desorber for removing carbon dioxide from process effluents

The carbon dioxide content of a process effluent flowing at a rate of 4 \( m^3/h \) is to be reduced from 1.5 kg/m\(^3\) to 0.02 mol/m\(^3\) by desorption with air in a packed stripper column operating at 20 °C and 1 bar. The carbon dioxide content of the inlet air is 0.03 % vol., and that of the air leaving the desorber outlet must not exceed 1 % vol. The desorption column is to be packed with 35-mm plastics Pall rings. Determine its main dimensions.

**Solution**

Average properties of the gas phase

- \( \mu_V = 28.96 \text{ kg/kmol} \)
- \( Q_V = 1.188 \text{ kg/m}^3 \)
- \( \eta_V = 17.98 \cdot 10^{-6} \text{ kg/ms} \)
- \( D_V = 15.4 \cdot 10^{-6} \text{ m}^2/\text{s} \)

Data for carbon dioxide

- \( \mu_A = 44.01 \text{ kg/kmol} \)
- \( Q_A = 1.818 \text{ kg/m}^3 \)

Average properties of the liquid phase

- \( \mu_L = 18.02 \text{ kg/kmol} \)
- \( Q_L = 998 \text{ kg/m}^3 \)
- \( \eta_L = 0.998 \cdot 10^{-3} \text{ kg/ms} \)
- \( D_L = 1.82 \cdot 10^{-9} \text{ m}^2/\text{s} \)
- \( \sigma_L = 72 \cdot 10^{-3} \text{ kg/s}^2 \)

Slope of equilibrium curve for the carbon dioxide-water/air system

\[ m_{Y,X} = 1440 \]

Characteristic data for the packing derived from Tables 4.1, 4.3, 4.6 and 5.1

- \( a = 145 \text{ m}^2/\text{m}^3 \)
- \( C_s = 2.654 \)
- \( C_P = 0.927 \)
- \( \varepsilon = 0.906 \text{ m}^3/\text{m}^3 \)
- \( C_l = 0.856 \)
- \( C_H = 0.718 \)
- \( C_V = 0.380 \)
Examples for the design of packed columns

Product streams corresponding to Fig. 14.2, right

\[ \dot{L}_o = 4 \cdot 998 = 3992 \text{ kg/h} \]
\[ = 4 \cdot \frac{998}{18.02} = 221.53 \text{ kmol/h} \]
\[ X_o = 1.5 \cdot \frac{18.02}{44.01 \cdot 998} = 6.154 \cdot 10^{-4} \text{ kmol/kmol} \]
\[ X_o = \frac{6.154 \cdot 10^{-4}}{1 + 6.154 \cdot 10^{-4}} = 6.150 \cdot 10^{-4} \text{ kmol/kmol} \]
\[ L = 221.53 (1 - 6.150 \cdot 10^{-4}) = 221.39 \text{ kmol/h} \]
\[ y_u = 0.03 \cdot 10^{-2} \cdot \frac{28.96 \cdot 1.818}{44.01 \cdot 1.188} = 3.02 \cdot 10^{-4} \text{ kmol/kmol} \]
\[ Y_u = \frac{3.02 \cdot 10^{-4}}{1 - 3.02 \cdot 10^{-4}} = 3.02 \cdot 10^{-4} \text{ kmol/kmol} \]
\[ y_o = 0.01 \cdot \frac{28.96 \cdot 1.818}{44.01 \cdot 1.188} = 0.01007 \text{ kmol/kmol} \]
\[ Y_o = \frac{0.01007}{1 - 0.01007} = 0.01017 \text{ kmol/kmol} \]
\[ X_a = 0.02 \cdot 10^{-3} \cdot \frac{18.02}{998} = 3.614 \cdot 10^{-7} \text{ kmol/kmol} \]

Total mass balance

\[ V = 221.39 \cdot \frac{6.154 \cdot 10^{-4} - 3.614 \cdot 10^{-7}}{0.01017 - 3.02 \cdot 10^{-4}} = 13.80 \text{ kmol/h} \]
\[ \dot{V}_o = 13.80 (1 + 0.01017) = 13.94 \text{ kmol/h} \]
\[ = 13.94 \cdot 28.96 = 403.71 \text{ kg/h} \]
\[ \dot{V}_u = 13.80 \cdot 28.96 (1 + 3.02 \cdot 10^{-4}) = 399.77 \text{ kg/h} \]
\[ = \frac{399.77}{1.188} = 336.50 \text{ m}^3/\text{h} \]
\[ \dot{L}_u = 221.39 \cdot 18.02 (1 + 3.614 \cdot 10^{-7}) = 3989 \text{ kg/h} \]

Resistance coefficient (Eqn (4-29))

\[ \xi_s = \frac{9.806}{2.654^2 \left[ \frac{3989.45}{998} \right]^{1/2} \left[ \frac{0.998 \cdot 10^{-3} \cdot 0.4}{17.98 \cdot 10^{-6}} \right]^{-0.652} = 1.980 \]
14.5 Design of a desorber for removing carbon dioxide from process effluents

Vapour velocity at the loading point \( \{\text{Eqn 4-28}\} \)

\[
u_{V,S} = \left( \frac{9.806}{1.980} \right)^{1/2} \left[ \frac{0.906}{145^{1/6}} - 145^{1/2} \left( \frac{12}{9.806} \frac{0.998 \cdot 10^{-3} u_L}{998} \right)^{1/3} \right].
\]

Liquid velocity at the loading point \( \{\text{Eqn (11-82)}\} \) for \( u_L \) and \( u_{V,S} \)

\[
u_L = \frac{3989.45}{399.77} \cdot \frac{1.188}{998} u_{V,S}
\]

Gas velocity at the loading point determined by solving Eqns (4-28) and (11-82) by iteration

\[
u_{V,S} = 1.195 \text{ m/s}
\]

Cross-sectional area of column \( \{\text{Eqn (14-26)}\} \)

\[
A_S = \frac{399.77/3600}{1.188 \cdot 1.195} = 0.0782 \text{ m}^2
\]

Column diameter \( \{\text{Eqn (14-27)}\} \)

\[
d_s = \sqrt{\frac{4}{\pi} \cdot 0.0739} = 0.316 \text{ m}
\]

Column diameter selected for planning: \( d_s = 0.32 \text{ m} \)

Cross-sectional area of planned column

\[
A_S = \frac{\pi}{4} \cdot 0.32^2 = 0.0804 \text{ m}^2
\]

Vapour and liquid loads in the planned column

\[
u_V = \frac{399.77/3600}{1.188 \cdot 0.0804} = 1.163 \text{ m/s}
\]

\[
u_L = \frac{3989.45/3600}{998 \cdot 0.0804} = 13.8 \cdot 10^{-3} \text{ m}^3/\text{m}^2\text{s}
\]

Theoretical liquid holdup \( \{\text{Eqn (4-27)}\} \)

\[
h_L = \left( \frac{12 \cdot 0.998 \cdot 10^{-3} \cdot 13.8 \cdot 10^{-3} \cdot 145^2}{9.806 \cdot 998} \right)^{1/3} = 0.0708 \text{ m}^3/\text{m}^3
\]
Particle diameter (Eqn (4-66))

\[ d_p = 6 \cdot \frac{1 - 0.906}{145} = 3.890 \cdot 10^{-3} \text{ m} \]

Wall factor (Eqn 4-57)

\[ f_s = \left(1 + \frac{4}{145 \cdot 0.32}\right)^{-1} = 0.9206 \]

Reynolds number for gas flow (Eqn (6-65))

\[ Re_g = \frac{1.163 \cdot 3.890 \cdot 10^{-3} \cdot 1.188}{(1 - 0.906) 17.98 \cdot 10^{-6}} \cdot 0.9206 = 2927 \]

Reynolds number for liquid flow (Eqn (4-69))

\[ Re_L = \frac{13.8 \cdot 10^{-3} \cdot 998}{145 \cdot 0.998 \cdot 10^{-3}} = 95.17 \]

Wetting factor for the packing (Eqn (4-68))

\[ W = \exp\left(\frac{95.17}{200}\right) = 1.61 \]

Resistance factor in gas flow (Eqn (4-67))

\[ \xi_L = 0.927 \cdot 1.61 \left(\frac{64}{2927} + \frac{1.8}{2927^{0.08}}\right) \left(\frac{0.906 - 0.0708}{0.906}\right)^{1.5} = 1.284 \]

Pressure drop per unit height in gas flow (Eqn (4-58))

\[ \frac{\Delta p}{H} = 1.284 \cdot \frac{145}{(0.906 - 0.0708)^3} \cdot \frac{1.188 \cdot 1.163^2}{2} \cdot \frac{1}{0.9206} = 240 \text{ Pa/m} \]

Hydraulic diameter (Eqn (5-15))

\[ d_h = 4 \cdot \frac{0.906}{145} = 0.02499 \text{ m} \]

Phase contact area (Eqn (5-16))

\[
\frac{a_{ph}}{a} = 3 \cdot 0.906^{0.5} \left(\frac{13.8 \cdot 10^{-3} \cdot 998}{145 \cdot 0.998 \cdot 10^{-3}}\right)^{-0.2} \cdot \left(\frac{(13.8 \cdot 10^{-3})^2 \cdot 998}{0.072 \cdot 145}\right)^{-0.75} \left(\frac{(13.8 \cdot 10^{-3})^2 \cdot 9.806}{145 \cdot 9.806}\right)^{0.45} = 0.799
\]
14.5  Design of a desorber for removing carbon dioxide from process effluents

Height of transfer unit in the liquid phase {Eqn (5-5)}

\[ HTU_L = \frac{1}{0.856} \left( \frac{0.998 \cdot 10^{-3}}{998 \cdot 9.806} \right)^{\frac{1}{16}} \left( \frac{0.02499}{1.82 \cdot 10^{-9}} \right)^{\frac{1}{12}} \left( \frac{13.8 \cdot 10^{-3}}{145} \right)^{\frac{2}{3}} \left( \frac{1}{0.799} \right) \]

= 0.772 m

Height of a transfer unit in the gas phase {Eqn (5-14)}

\[ HTU_V = \frac{1}{0.380} \left( \frac{0.906 - 0.0708}{145^{3/2}} \cdot \frac{0.02499^{1/2}}{15.4 \cdot 10^{-6}} \cdot \frac{1.163}{1.188} \right) \left( \frac{15.4 \cdot 10^{-6} \cdot 1.188}{17.98 \cdot 10^{-6}} \right)^{1/3} \left( \frac{1}{0.799} \right) = 0.187 m \]

Differences in gas loads {Eqns (14-16) and (14-17)}

\[ \Delta X_o = 6.154 \cdot 10^{-4} - \frac{0.01017}{1440} = 6.083 \cdot 10^{-4} \text{ kmol/kmol} \]

\[ \Delta X_u = 3.614 \cdot 10^{-7} - \frac{3.02 \cdot 10^{-4}}{1440} = 1.516 \cdot 10^{-7} \text{ kmol/kmol} \]

Number of liquid-side transfer units {Eqn (14-15)}

\[ NTU_{OL} = \frac{6.154 \cdot 10^{-4} - 3.614 \cdot 10^{-7}}{6.083 \cdot 10^{-4} - 1.516 \cdot 10^{-7}} \cdot \ln \left( \frac{6.083 \cdot 10^{-4}}{1.516 \cdot 10^{-7}} \right) = 8.39 \]

Stripping factor {Eqn (1-68)}

\[ \lambda = 1440 \cdot \frac{13.80 \text{ kmol/h}}{221.39 \text{ kmol/h}} = 89.79 \]

Height of an overall transfer unit {Eqn (1-63)}

\[ HTU_{OL} = 0.772 + \frac{1}{89.79} \cdot 0.187 = 0.774 \text{ m} \]

Height of bed if end and distribution effects are neglected {Eqn 1-65)}

\[ H = 0.774 \cdot 8.39 = 6.494 \text{ m} \]
14.6 Determination of the difference between the theoretical and the real liquid holdup

The purpose here is to determine the differences between the theoretical and the real liquid holdups applicable to the loading conditions under which the columns discussed in Sections 14.4 and 14.5 were operated.

Solution

Known values in Section 14.4

\[ C_h = 0.784 \quad Q_L = 997 \text{ kg/m}^3 \quad u_L = 4.55 \cdot 10^{-3} \text{ m}^3/\text{m}^2/\text{s} \]
\[ a = 110 \text{ m}^2/\text{m}^3 \quad \eta_L = 0.857 \cdot 10^{-3} \text{ kg/ms} \quad Re_L = 48.12 \]

Froude number for the liquid phase \{Eqn (4-27)\}

\[ Fr_L = \frac{110 (4.55 \cdot 10^{-3})^2}{9.806} = 2.322 \cdot 10^{-4} \]

Hydraulic area \{Eqn (4-75)\}

\[ a_h \cdot a = 0.85 \cdot 0.784 \cdot 48.1^{0.25} (2.322 \cdot 10^{-4})^{0.1} = 0.760 \]

Real liquid holdup \{Eqn (4-73)\}

\[ h_L = \left( \frac{12 \cdot 0.857 \cdot 10^{-3} \cdot 110^2 \cdot 4.55 \cdot 10^{-3}}{997 \cdot 9.806} \right)^{1/3} \cdot 0.760^{2/3} \]
\[ = 0.0322 \text{ m}^3/\text{m}^3 \]

Calculated difference between the theoretical and real holdups

\[ \frac{\Delta h_L}{h_L} = \frac{0.0387 - 0.0322}{0.0387} = 0.168 \]

Known values in Section 14.5

\[ C_h = 0.718 \quad Q_L = 998 \text{ kg/m}^3 \quad u_L = 13.8 \cdot 10^{-3} \text{ m}^3/\text{m}^2/\text{s} \]
\[ a = 145 \text{ m}^2/\text{m}^3 \quad \eta_L = 0.998 \cdot 10^{-3} \text{ kg/ms} \quad Re_L = 95.17 \]

Froude number for the liquid phase \{Eqn (4-27)\}

\[ Fr_L = \frac{145 (13.8 \cdot 10^{-3})^2}{9.806} = 2.816 \cdot 10^{-3} \]
Hydraulic area \( \text{Eqn (4-75)} \):

\[
\frac{a_h}{a} = 0.85 \cdot 0.718 \cdot 95.17^{0.25} \cdot (2.816 \cdot 10^{-3})^{0.1} = 1.060
\]

Real liquid holdup \( \text{Eqn (4-73)} \):

\[
h_L = \left( \frac{12 \cdot 0.998 \cdot 10^{-3} \cdot 145^2 \cdot 13.8 \cdot 10^{-3}}{998 \cdot 9.806} \right)^{1/3} \cdot 1.060^{2/3} = 0.0736 \text{ m}^3/\text{m}^3
\]

Calculated difference between the theoretical and real holdups:

\[
\frac{\Delta h_L}{h_L} = \frac{0.0708 - 0.0736}{0.0708} = -0.039
\]

14.7 Additional column height required to compensate maldistribution

In a column of \( d_s = 2250 \text{ mm} \) diameter packed with 50-mm Pall rings, the liquid distributor gives rise to maldistribution of \( M = 10\% \). The phase ratio is \( L/V = 1 \). If the maldistribution were to remain effective over the entire height of the column, what would be the additional column height required to compensate the attendant loss in efficiency?

Solution

The loss in efficiency can be read off from the diagram in Fig. 8.12 against the value

\[
\frac{d_s}{d} = \frac{2250}{50} = 45
\]

The numerical value thus obtained is

\[
\Delta E = 15\%
\]

In other words, the efficiency would be reduced by a factor \( \text{Eqn (8-24)} \)

\[
E_C = 1 - \frac{15}{100} = 0.85
\]

Hence, the relative increase in height required to compensate the maldistribution is given by

\[
\frac{\Delta H}{H} = \frac{1}{E_C} - 1 = \frac{1}{0.85} - 1 = 0.176
\]
14.8 Retrofitting a plate column with packing for steam distillation of crude oil

In order to avoid thermal decomposition, the partial pressure of the hydrocarbon vapour must not be allowed to exceed 34 mbar at the lower end of the intermediate zone in a vacuum column \( (p_T = 100 \text{ mbar}) \) designed for fractionating crude oil by steam rectification. The plates previously used (subscript 2) gave rise to a pressure drop of 110 mbar between the head of the column and the intermediate zone. After the column had been retrofitted with low-pressure-drop packing (subscript 1), the pressure drop was only 70 mbar. How much steam has thus been saved?

Solution

According to Eqn (12-50), the pressure drop after retrofitting is given by

\[
p_{11} - p_T = (i - 1) \left( \frac{\Delta p}{n_i} \right)_1 = 70 \text{ mbar}
\]

The pressure drop before retrofitting is given by

\[
p_{12} - p_T = (i - 1) \left( \frac{\Delta p}{n_i} \right)_2 = 110 \text{ mbar}
\]

According to Eqn (12-63), the factor \( k \) for the difference in pressure drop is given by

\[
k = \left( \frac{\Delta p}{n_i} \right)_2 \frac{\Delta p}{n_i}_1 \frac{110}{70} = 1.57
\]

The relative savings in steam achieved by the installation of packing follows from Eqn (12-67), i.e.

\[
\frac{\Delta S}{S_2} = \frac{1.57 - 1}{1.57 + \frac{100 - 34}{70}} = 0.226
\]

The volume flow rate of steam per unit flow rate of the hydrocarbon vapours after retrofitting is obtained from Eqn (12-64), i.e.

\[
\frac{\dot{S}_1}{\dot{V}} = \frac{100 + 70 - 34}{34} = 4
\]

The corresponding flow rate before retrofitting is obtained from Eqn (12-65), i.e.

\[
\frac{\dot{S}_2}{\dot{V}} = \frac{100 + 110 - 34}{34} = 5.17
\]
Therefore, the savings in steam per unit flow rate of product vapour is as follows:

\[
\frac{\Delta \dot{S}}{\dot{V}} = \frac{\dot{S}_2 - \dot{S}_1}{\dot{V}} = 5.17 - 4 = 1.17
\]

The same figure can be obtained from Eqn (12-66), i.e.

\[
\frac{\Delta \dot{S}}{\dot{V}} = \frac{1}{34} \cdot 70 (1.57 - 1) = 1.17
\]

The savings in steam achieved by substituting packing for fractionating plates can be determined from the values obtained for $\Delta \dot{S}/\dot{V}$ and $\dot{S}_2/\dot{V}$, i.e.

\[
\frac{\Delta \dot{S}}{\dot{S}_2} \cdot 100 = \frac{1.17}{5.17} \cdot 100 = 22.6\%
\]

### 14.9 Reduction in height effected by redistributors

It is intended to separate a mixture in a packed column with 30 transfer units, 35-mm metallic Pall rings, and optimum inlet distribution. How much height would be saved by installing three redistributors?

**Solution**

Assume that the inlet effect and the data describing the efficiency are identical to those listed in Table 9.1. In this case, the effective height of the column without redistributors is given by Eqn (9-20), i.e.

\[
(H_T)_{n = 0} = [30 - (0.5 + 1.9)] \cdot \frac{1}{2.125} = 13 \text{ m}
\]

The relative reduction in the height of the bed can be read off against $n_r = 3$ in the upper diagram shown in Fig. 9.7. Thus,

\[
\Delta H / H_T = 0.26
\]

The absolute reduction in height is therefore

\[
\Delta H = 0.26 \cdot 13 = 3.4 \text{ m}
\]

Hence, the total effective height of the bed with three redistributors is

\[
(H_T)_{n_r = 3} = 13 - 3.4 = 9.6 \text{ m}
\]
and the effective height between the redistributors is

\[ H_r = \frac{9.6}{3 + 1} = 2.4 \text{ m} \]

This value can also be read off on the axis of abscissae against \( n_r = 3 \) in Fig. 9.7.

Assume further that the height occupied by a redistributor in the column is \( h_r = 0.4 \text{ m} \). According to Eqn (9-50), the relative reduction in the total height would then be

\[ \left( \frac{\Delta H}{H_T} \right)_5 = 1 - \frac{1}{13} [2.4 \times (3 + 1) + 3 \times 0.4] = 0.169 \]

The absolute saving in the height of the column shell is therefore

\[ \Delta H_5 = 0.169 \times 13 = 2.2 \text{ m} \]

14.10 Advantages of geometrically arranged packing in fractionating fatty acids

A modern plant for processing palm-kernel oil consists of nine distillation stages in tandem. The plans for process optimization included the improvement of product quality by an appropriate choice of column internals. A Hermann Stage method was adopted for the design.

Solution

Since the feedstock is sensitive to heat, the distillation stages were designed with structured packing and falling-film evaporators. As a consequence, the residence times were very short, and the bottom temperatures were comparatively low (Fig. 14.5). The effects on the product quality were as follows.

The overhead product in each stage ranging from Column 2 (octanoic acid) to Column 5 (myristic acid) is the pure distillate of the unhydrogenated crude acid concerned. Its purity is higher than 99% during continuous operation and even higher than 99.5% for lauric (C12) and myristic (C14) acids. Owing to the unsaturated C16 fraction, the purity of the palmitic acid in the sidestream distillate of Column 8 is only just higher than 97%. Since the linoleic acid fraction (C18\(\)) in the feed is higher than that of the oleic acid (C18\(\)), the sidestream distillate in the next column downstream (Column 9) contains only 85% of oleic acid (C18H); the remainder consists of 8–10% of linoleic acid (C18\(\)) and 4–5% of stearic acid. If the trans-fatty acid content is less than 0.5%, the oleic acid content in this fraction is higher than that attained in the Henkel or Alpha-Laval detergent processes.

The flow chart shown in Fig. 14.5 for a fatty acid fractionating plant includes the continuous cracker, the glycerol-water evaporation unit, and the pure glycerol distillation column. The two latter also incorporate a Hermann Stage process and the associated engineering. The entire plant has been on stream in Malaysia since 1992.
A production effluent containing 0.241 kmol/m$^3$ of acetone is to be purified by liquid-liquid extraction with toluene. The packed column is to be designed and operated for 80% acetone recovery. The intended packing is 25-mm Bialecki rings. The column diameter and the effective height of bed should permit the effluent to flow at a rate of $1557$ m$^3$/h, the column load may not exceed 35% of that at the flood point, and the extractant is toluene flowing at a rate of $2829$ m$^3$/h.
Solution

Characteristic data for systematically stacked Bialecki rings (cf. Table 13.2)

\[ \varepsilon = 0.928 \quad C_{FL} = 0.67 \]
\[ l = 0.050 \text{ m} \quad C_h = 2.151 \]
\[ C_d = 1 \]

Physical properties of the system

\[ \varrho_D = 857 \text{ kg/m}^3 \quad \varrho_C = 998 \text{ kg/m}^3 \]
\[ \eta_D = 0.55 \cdot 10^{-3} \text{ kg/ms} \quad \eta_C = 0.893 \cdot 10^{-3} \text{ kg/ms} \]
\[ D_D = 2.478 \cdot 10^{-9} \text{ m}^2/\text{s} \quad D_C = 1.3 \cdot 10^{-9} \text{ m}^2/\text{s} \]
\[ \sigma = 0.026 \text{ kg/s}^2 \quad \mu = 0.64 \]

Phase flow ratio

\[ \frac{u_D}{u_C} = \frac{2.829}{1.557} = 1.82 \]

Dispersed phase holdup \{Eqn (13-20)\}

\[ h_{D,FI} = \frac{(1.82^2 + 8 \cdot 1.82)^{1/2} - 3 \cdot 1.82}{4 (1 - 1.82)} = 0.375 \text{ m}^3/\text{m}^3 \]

Droplet velocity \{Eqn (13-3)\}

\[ u = \left[ 4 \cdot \frac{9.81 (998 - 857) 0.026}{998^2} \right]^{1/4} = 0.1096 \text{ m/s} \]

Velocity of continuous phase at the flood point \{Eqn (13-16)\}

\[ u_{C,FI} = 0.67 \cdot 0.928 \cdot 0.1096 (1-0.375)^2 (1 - 2 \cdot 0.375) = 6.6 \cdot 10^{-3} \text{ m}^3/\text{m}^3 \]

Velocity of continuous phase under operating conditions

\[ u_C = 0.35 \cdot 6.6 \cdot 10^{-3} = 2.31 \cdot 10^{-3} \text{ m}^3/\text{m}^3 \]

Diameter of extraction column \{Eqn (14-35)\}

\[ d_s = \sqrt[4]{\frac{4 \cdot 1.557/3600}{\pi \cdot 2.31 \cdot 10^{-3}}} = 0.488 \text{ m} \]

Velocity of dispersed phase under operating conditions

\[ u_D = 1.82 \cdot 2.31 \cdot 10^{-3} = 4.21 \cdot 10^{-3} \text{ m}^3/\text{m}^3 \]
Dispersed phase holdup under operating conditions \{(13-11)\}

\[
h_D = \frac{2.151}{0.928} \cdot \frac{4.21 \cdot 10^{-3}}{0.1096} = 0.0890 \text{ m}^3/\text{m}^3
\]

Mass transfer constant \{(13-39)\}

\[
C_m = \frac{5.69}{1} \sqrt{\frac{2.151}{0.050}} = 37.32
\]

Height of overall transfer unit \{(13-47)\}

\[
HTU_{OC} = \frac{1}{37.32} \cdot \frac{0.026^{5/8}}{[9.81 (998 - 857)]^{3/8} \cdot 998^{1/4} \cdot \frac{1}{1.82}} \cdot \left( \frac{1}{(1.3 \cdot 10^{-9})^{1/2}} + \frac{1}{0.64} \cdot \frac{1}{(2.478 \cdot 10^{-9})^{1/2}} \cdot \frac{998}{857} \right) = 1.14 \text{ m}
\]

Concentration of continuous phase after extraction \{(14-33)\}

\[
c_{c_{u'}} = (1 - 0.8) \cdot 0.241 = 0.0482 \text{ kmol/m}^3
\]

Concentration of solvent after extraction \{(14-34)\}

\[
c_{D_{o}} = \frac{1}{1.82} (0.241 - 0.0482) + 0 = 0.1059 \text{ kmol/m}^3
\]

Driving concentration difference at the head of the column \{(14-30)\}

\[
\Delta c_o = 0.241 - \frac{1}{0.64} \cdot 0.1059 = 0.0755 \text{ kmol/m}^3
\]

Driving concentration difference at the foot of the column \{(14-31)\}

\[
\Delta c_u = 0.0482 - \frac{1}{0.64} \cdot 0 = 0.0482 \text{ kmol/m}^3
\]

Number of overall transfer units in the continuous phase \{(14-29)\}

\[
NTU_{OC} = \frac{0.241 - 0.0482}{0.0755 - 0.0482} \cdot \ln \frac{0.0755}{0.0482} = 3.17
\]

Effective height of bed \{(13-22)\}

\[
H = 1.14 \cdot 3.17 = 3.61 \text{ m}
\]