Numerical solution of steady viscous flow of a micropolar fluid driven by injection between two porous disks

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Abstract

Steady, laminar, incompressible and two-dimensional flow of a micropolar fluid between two porous coaxial disks is considered. To describe the working fluid, we use the micropolar model given by Eringen. The governing equations of motion are reduced to a set of non-linear coupled ordinary differential equations using Berman’s similarity transformation. The SOR iterative method is used to solve the differential equations numerically. For higher order accuracy, the results obtained are further refined and enhanced by Richardson’s extrapolation method. The results of micropolar fluids are compared on different grid sizes.

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Keywords: Porous disks; Micropolar; Similarity transformation; Extrapolation; SOR method; Microrotation

1. Introduction

Different authors have considered the effects of porous boundaries on steady, laminar and incompressible flow for different geometrical shapes. Two-dimensional flows in a porous channel has been studied by several authors, e.g. Berman [1], Cox [2], Brady [3], Terrill and Shrestha [4,5], Terrill [6], and Robinson [7]. Berman [1] considered flow which was driven by suction or injection, while Cox [2] examined by considering channel with one porous wall and other non-porous but was accelerating. Terrill and Shrestha [5] considered the laminar flow for small Reynolds numbers through parallel and uniformly porous walls of different permeability with different suction and injection normal velocities at the walls. They obtained the series solution using perturbation method and compared it with the numerical solution by calculating the skin friction at the lower and the upper wall. Terrill [6] obtained the complete solution of the laminar flow of a fluid in a uniformly porous channel with large injection by the method of inner and outer expansions and included the viscous layer. The resulting series solutions were confirmed by numerical results. In all these cases the Navier–Stokes

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equations are reduced to ordinary non-linear differential equations of third order for which approximate solutions are obtained by a mixture of analytical and numerical methods.

The problem of disk flows has constituted a major field of study in fluid mechanics. These flows have immediate technical applications in the fields of rotating machinery, computer storage devices, viscometry, lubrication, crystal growth processes, heat and mass exchangers, biomechanics and oceanography. Elcrat [8] has proved the theorem of existence and uniqueness for non-rotational fluid motion between fixed porous disks with arbitrary uniform injection or suction. Rasmussen [9] examined the steady viscous flow between two porous disks. The flow was driven by injection or suction at both walls. He used an extension of Berman’s [1] similarity transformation to reduce governing equations to a set of non-linear coupled ordinary differential equations in dimensionless form. The boundary value problem was converted into initial value problem and shooting method was used to solve it numerically. He only considered the symmetric case in which there was equal blowing or suction at both the disks. All the above researchers have done their work for Newtonian fluids.

Eringen [10] introduced micropolar fluids which is a subclass of microfluids. The micropolar fluids are those which consist of bar-like elements. For example, the micropolar model has been used to describe the flow of liquid crystals which are made up of dumbbell molecules. Other examples of such fluids are polymeric fluids and real fluids with suspensions, fluids with some polymeric additives, suspension solutions, and nematogenic and smectogenic liquid crystals. The micropolar fluids have applications in blood flow, lubricants, porous media, turbulent shear flows, and flow in capillaries and microchannels. The concept of such fluids is to provide a mathematical model for the behavior of fluids taking into account the initial characteristics of the substructure particles which are allowed to undergo rotation. Guram and Anwar [11] considered the problem of steady, laminar and incompressible flow of a micropolar fluid due to a rotating disc with uniform suction and injection. The equations of motion were reduced to dimensionless form and then solved by Gauss–Siedel iterative procedure with Simpson’s rule. Kelson et al. [12] analyzed self-similar boundary layer flow of a micropolar fluid in a porous channel. The flow was driven by uniform mass transfer through the channel walls. They used Berman [1] type similarity transformation to reduce governing equations of motion to a set of non-linear coupled ordinary differential equations which were then solved for large mass transfer via a perturbation analysis. Anwar Kamal and Siffat Hussain [13] examined the steady, incompressible and laminar flow of micropolar fluids inside an infinite channel. The flow was driven due to a surface velocity proportional to the streamwise coordinates. The governing equations were reduced to non-linear ordinary differential equations by using similarity transformation. These equations were then solved using numerical procedures which include SOR method. Anwar Kamal and Siffat Hussain [14] predicted the three-dimensional micropolar fluid motion caused by the stretching of a surface. The resulting ordinary differential equations of motion were solved numerically using SOR method.

In this paper, we consider the steady, laminar and incompressible flow of a micropolar fluid between two stationary coaxial porous disks. We neglect the effects of body force and body couple. The flow is symmetrically driven by equal injection through the two porous disks. We use SOR method to solve the governing equations of motion. The Richardson’s extrapolation method is used to obtain higher order accuracy. The basic analysis and formulation of the problem are given in Section 2. Section 3 contains the detail of numerical scheme which we used while Section 4 explains the procedure adopted during numerical computation. Finally the results are discussed and presented in tabular as well as graphical form in Section 5.

2. Basic analysis

For the problem under consideration, the suitable coordinates will be cylindrical polar coordinate system. Consider two stationary porous disks of radius $R_O$ located in the $z = -L$ and $z = L$ planes, respectively, and let the centres of the disks coincide with the axis $r = 0$ as shown in Fig. 1.

The velocity components $u_r$ and $u_z$ are taken to be in the direction of $r$- and $z$-axes, respectively. Fluid is injected at both the disks with constant velocities $V_1$ and $V_2$ at the lower and upper disk, respectively.
The governing equations of motion for the micropolar fluid given by Eringen [10] are as follows:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \] 

\[ (\lambda + 2 \mu + \kappa) \nabla (\mathbf{V} \cdot \nabla \mathbf{V}) - (\mu + \kappa) \nabla \times \nabla \times \mathbf{V} + \kappa \nabla \times \mathbf{\omega} - \nabla \pi + \rho \mathbf{f} = \rho \mathbf{\dot{V}}, \]  

\[ (\alpha + \beta + \gamma) \nabla (\mathbf{\omega} \cdot \nabla \mathbf{\omega}) - \gamma (\nabla \times \nabla \times \mathbf{\omega}) + \kappa \nabla \times \mathbf{V} - 2 \kappa \mathbf{\omega} + \rho \mathbf{l} = \rho \mathbf{l}, \] 

where \( \mathbf{V} \) is the fluid velocity vector, \( \mathbf{\omega} \) the microrotation, \( \rho \) the density, \( \pi \) the pressure, \( \mathbf{f} \) and \( \mathbf{l} \) are body force and body-couple per unit mass, respectively, \( j \) the microinertia, \( \lambda, \mu, \alpha, \beta, \gamma, \kappa \), the material constants (or viscosity coefficients), where dot signifies material derivative. Here the velocity vector \( \mathbf{V} \) and \( \mathbf{\omega} \) the microrotation vector are unknown.

The velocity and the spin rotation components are

\[ \mathbf{V} = (u(r,z), 0, w(r,z)), \] 

\[ \mathbf{\omega} = (0, \Phi(r,z), 0). \] 

Using (2.4) in the governing equations of motion (2.1)–(2.3), we found

\[ (\mu + \kappa)(\partial^2 u/\partial r^2 + (1/r)\partial u/\partial r - u/r^2 + \partial^2 u/\partial z^2) - \kappa \partial \Phi/\partial z - \partial \pi/\partial r \]

\[ = \rho(\partial u/\partial r + w \partial u/\partial z), \] 

\[ (\mu + \kappa)(\partial^2 w/\partial r^2 + (1/r)\partial w/\partial r + \partial^2 w/\partial z^2) + \kappa(\partial \Phi/\partial r + \Phi/\partial r) - \partial \pi/\partial z \]

\[ = \rho(\partial w/\partial r + w \partial w/\partial z), \] 

\[ \gamma(\partial^3 \Phi/\partial r^3 + (1/r)\partial \Phi/\partial r - \partial \Phi/r^2 + \partial^2 \Phi/\partial z^2) + \kappa(\partial u/\partial z - \partial w/\partial r) - 2 \kappa \Phi \]

\[ = \rho j(\partial \Phi/\partial r + w \partial \Phi/\partial z) \] 

\[ \partial u/\partial r + u/r + \partial w/\partial z = 0. \]

On the disks the axial velocity \( u_z \) is prescribed and the radial velocity \( u_r \) is zero. Thus we have the following boundary conditions:

\[ u_z(r, -L) = 2V_1, \quad u_z(r, L) = 2V_2, \quad u_r(R, -L) = 0, \quad u_r(R, L) = 0, \] 

where \( V_1 \) and \( V_2 \) are constants independent of \( r \) and \( \theta \).

We have to solve Eqs. (2.5)–(2.8) subject to boundary conditions (2.9). We let

\[ u_r = -rF(z), \quad u_z = 2F(z), \quad \Phi = -rG. \] 

If we use (2.10) in (2.8), then we see that equation of continuity is identically satisfied. Using (2.10) in (2.5) and (2.6) we get, after some simplifications and eliminating pressure term, the following equation:

\[ (\mu + \kappa)F^{(iv)} - \kappa G'' - 2 \rho FF'' = 0, \] 

where \( \partial^4 \pi/\partial r \partial z = 0. \)

Now using (2.10) in (2.7) we found that

\[ \gamma G'' + \kappa F'' - 2 \kappa G - \rho j(F' G - 2FG') = 0. \]
Next dimensionless variables may be defined as
\[ f(\lambda) = \frac{F(Z)}{V}, \quad g(\lambda) = \frac{L^2 G(Z)}{V}, \]  
(2.14)
where \( \lambda = Z/L \) and \( V \) is the larger of \( V_1 \) and \( V_2 \).

Using (2.14) in (2.11) and (2.13) we shall find that
\[
\begin{align*}
&f''(\lambda) - c_1 g'' \equiv -2R^2 f'' = 0, \\
&g'' + c_2(f'' - 2g) - c_3(f'g - 2fg') = 0,
\end{align*}
\]
(2.15)
(2.16)
where \( R = \rho VL/(\mu + \kappa) \) is the Reynolds number and
\[ c_1 = \kappa/(\mu + \kappa), \quad c_2 = \kappa L^2/\gamma, \quad c_3 = \rho jLV/\gamma \]
are dimensionless constants.

Integrating (2.15) with respect to \( \lambda \) we get
\[
f'' - c_1 g' - 2R^2 f'' + R^2 \beta = \beta,
\]
(2.17)
where \( \beta \) is constant of integration and is known as pressure constant.

Using (2.14) in (2.10) the boundary conditions reduce to
\[
\begin{align*}
f(1) &= 1, \quad f(-1) = -1, \\
f'(1) &= 0, \quad f'(-1) = 0, \\
g(1) &= 0, \quad g(-1) = 0.
\end{align*}
\]
(2.18)
We have to solve (2.16) and (2.17) subject to boundary conditions (2.18). We note that (2.16) and (2.17) reduce to the corresponding equations for a Newtonian fluid for vanishing microrotation and \( \kappa = 0 \). Eqs. (2.16) and (2.17) being non-linear could not be solved analytically and hence we solve them by using suitable numerical technique.

3. Finite difference equations

Let \( p = f'' = \frac{df}{d\lambda} \)
(3.1)
then (2.16) and (2.17) become,
\[
\begin{align*}
p'' - c_1 g' - 2Rf p' + R p^2 &= \beta, \\
g'' + c_2(p' - 2g) - c_3(pg - 2fg') &= 0,
\end{align*}
\]
(3.2)
(3.3)
with the boundary conditions (2.18) as follows:
\[
\begin{align*}
f(1) &= 1, \quad f(-1) = -1, \\
p(1) &= 0, \quad p(-1) = 0, \\
g(1) &= 0, \quad g(-1) = 0.
\end{align*}
\]
(3.4)
If we approximate the derivatives in (3.2) and (3.3) by central-difference approximations at a typical point \( \lambda = \lambda_n \) of the interval \([-1, 1]\), we obtain,
\[
\begin{align*}
&\left(2R^2 p_n - 4\right)p_n + (2 - 2Rh f_n)p_{n+1} + (2 + 2Rh f_n)p_{n-1} = 2h^2 \beta + c_1 h(g_{n+1} - g_{n-1}), \\
&(4 + 4c_2 h^2 + 2c_3 h^2 p_n)g_n = (2 + 2c_3 h f_n)g_{n+1} + (2 - 2c_3 h f_n)g_{n-1} + c_2 h(p_{n+1} - p_{n-1}),
\end{align*}
\]
(3.5)
(3.6)
where \( h \) represents the grid length and \( p_n = p(\lambda_n), \ g_n = g(\lambda_n) \) and \( f_n = f(\lambda_n) \).

4. Computational procedure

We now integrate numerically (3.1) and system of finite difference equations (3.5) and (3.6) at each required grid point of the interval \([-1, 1]\). Eq. (3.1) is integrated using Simpson’s rule Gerald [15, p. 67–69] with the
Iterative procedure: For a suitable choice of the values of the grid size \( h \) and the relaxation parameter \( \omega \), an iterative procedure is started with some initial guess for the values of the constant of integration \( \beta \) and the solution vectors \( p \), \( g \), and \( f \), where \( k \)th iteration performs the following steps:

I. Next approximation for the solution of the finite difference equations (3.5) and (3.6), \( p^{(k+1)} \) and \( g^{(k+1)} \), respectively, is generated by SOR method subject to the last four conditions in (3.4).

II. New approximation for the solution of the differential equation (3.1), \( f^{(k+1)} \), is computed by Simpson’s rule subject to the first boundary condition given in (3.4), where \( p^{(k+1)} \) is employed for \( p \) in Eq. (3.5).

III. \( p^{(k+1)} \), \( g^{(k+1)} \) and \( f^{(k+1)} \) are compared with \( p^{(k)} \), \( g^{(k)} \) and \( f^{(k)} \), respectively, to test for convergence.

The iterative procedure is stopped if the following criteria are satisfied for three consecutive iterations:

\[
\|p^{(k+1)} - p^{(k)}\|_{L^2} < \text{TOL}_{\text{iter}},
\]

\[
\|f^{(k+1)} - f^{(k)}\|_{L^2} < \text{TOL}_{\text{iter}},
\]

\[
\|g^{(k+1)} - g^{(k)}\|_{L^2} < \text{TOL}_{\text{iter}}.
\]

Finding constant of integration: The constant of integration \( \beta \) is determined by hit and trial method by requiring that the computed value of \( f \) at the upper boundary \( \lambda = 1 \) matches with the given boundary value of \( f \) up to a specified tolerance \( \text{TOL}_{\text{iter}} \).

Improving order of accuracy by extrapolation: To increase the order of accuracy, the discrete problem is first solved on a basic grid, say \( h \in [0, H] \) for \( H > 0 \). A sequence of approximate solutions is then computed on successively refined grids. Let \( U(h) \) denote the discrete solution corresponding to the step sizes

\[ h_l = \frac{H}{n_l}, \quad l = 1, 2, 3, \ldots, \]

where \( U(h) \) stands for either of \( p \) and \( f \) and \( \{n_l\} \) is a sequence of integers associated with the step size sequence \( \{h_l\} \) to govern the successive refinement procedure. There are several choices for the sequence \( \{n_l\} \) found in literature. We use the Romberg sequence given below

\[ S_R = \{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024\}. \]

On the basis of solutions \( U(h) \), higher order approximations to the exact solutions can be obtained by the use of Richardson’s extrapolation. This process can be carried out using any extrapolation scheme [19], e.g. polynomial extrapolation (Aitken–Neville algorithm) or rational extrapolation (Stoer’s algorithm). We use polynomial extrapolation, which can be presented as

\[
U_{l,1} = U(h_l), \quad l = 1, 2, 3, \ldots,
\]

\[
U_{l,m} = U_{l,m-1} + \frac{U_{l,m-1} - U_{l-1,m-1}}{(n_l/n_{l-1})^2 - 1}, \quad m = 2, 3, \ldots, l, \quad l \geq 2,
\]

where \( l \) indicates the grid level and \( m \) is the number of extrapolation steps. The order of approximation of the solution \( U_{l,m} \) increases in even multiples of \( m \), i.e. the error of \( U_{l,m} \) is proportional to \( H^{2m} \).

Improving initial guess for higher grid levels: An element of multigrid methods called nested iterations has been used to a limited extent to obtain an improved initial guess for the solution \( U^{(l)} \), where \( l > 1 \), so that rapid convergence of the iterative procedure may be achieved. This may be done by subjecting the solution \( U^{(l-1)} \) to an appropriate interpolation operator so that interpolated values are provided at the new grid points of the level \( l \). A computationally convenient choice is the linear operator which gives the improved initial guess for the solution \( U^{(l)} \) as follows:
\[ U_{2i}^{(l)} = U_{i}^{(l-1)}, \]
\[ U_{2i+1}^{(l)} = \frac{1}{2}(U_{i}^{(l-1)} + U_{i+1}^{(l-1)}), \]

where \( 0 \leq i \leq n_{l-1}N - 1 \), and \( l \) denotes the grid level.

5. Results and discussion

The physics of the problem under consideration is explained through the detailed interpretation of the axial and radial velocity distributions for various Reynolds number \( R \). The calculations were made on different grid sizes, namely, \( h = 0.01 \), \( h = 0.005 \), \( h = 0.0025 \). The extrapolated values are calculated at the finer grid size. The results at the three grid sizes and their extrapolated values are shown in Tables 1–6 for \( f(\lambda) \), \( f'(\lambda) \) and \( g(\lambda) \).

The shear stresses or skin friction at the lower and upper disk is also discussed. There is no boundary layer on the disk in case of injection at the disk. There is an internal viscous layer at the plane \( Z = 0 \). The position of this viscous or shear layer can be determined numerically. It is a point \( \dot{\lambda} = \lambda_0 \) where \( f(\lambda) = 0 \). In case of asymmetrical flow the position of this viscous layer changes upto the plane \( Z = 0 \) in the lower region between the two porous disks. (This would be explained in an other paper.) The skin friction at the upper and lower disk for micropolar fluid is shown in Table 7.

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<th>Extrapolated value</th>
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We conclude from Table 7 that the skin friction at the either disk slightly changes for large negative values of the Reynolds number $R$ while for $R$ closer to 0 and $R > 0$, a significant change is observed for small change in the value of $R$. 

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The axial velocity distribution of micropolar fluids for various $R$ is shown in Fig. 2. It helps to locate the position of the viscous layer which is generated due to large blowing at the two disks. We observe from Fig. 2 that the position of the viscous layer coincides with centre between the two disks in all cases of the Reynolds number. The radial velocity profiles are parabolic in all cases of $R$ discussed as shown in Fig. 3. The maximum value of the radial velocity increases and the graphs of parabola get sharper as we increase the blowing at the disks. The microrotation of the micropolar fluid for various $R$ is shown in Fig. 4. There is no spin or micro-rotation at the point $Z = 0$ and the concavity of the graph changes at this point. The spin or micro rotation profiles are similar for all cases of $R$.

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![Fig. 2. Axial velocity profiles of micropolar fluids for various $R$.](image)
References