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Journal Review Article

A review of techniques, advances and outstanding issues in numerical modelling for rock mechanics and rock engineering ☆

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Abstract

The purpose of this review paper is to present the techniques, advances, problems and likely future developments in numerical modelling for rock mechanics. Such modelling is essential for studying the fundamental processes occurring in rocks and for rock engineering design. The review begins by explaining the special nature of rock masses and the consequential difficulties when attempting to model their inherent characteristics of discontinuousness, anisotropy, inhomogeneity and inelasticity. The rock engineering design backdrop to the review is also presented. The different types of numerical models are outlined in Section 2, together with a discussion on how to obtain the necessary parameters for the models. There is also discussion on the value that is obtained from the modelling, especially the enhanced understanding of those mechanisms initiated by engineering perturbations. In Section 3, the largest section, states-of-the-art and advances associated with the main methods are presented in detail. In many cases, for the model to adequately represent the rock reality, it is necessary to incorporate couplings between the thermal, hydraulic and mechanical processes. The physical processes and the equations characterizing the coupled behaviour are included in Section 4, with an illustrative example and discussion on the likely future development of coupled models. Finally, in Section 5, the advances and outstanding issues in the subject are listed and in Section 6 there are specific recommendations concerning quality control, enhancing confidence in the models, and the potential future developments.

Keywords: Review; Rock mechanics; Numerical modelling; Design; Coupled processes; Outstanding issues

Contents

☆ This is the second of a series of Journal Review Articles commissioned by the Editor. The series consists of articles reviewing significant or topical subjects, or subjects requiring expert explanation. This Review is a significantly expanded version of the "Numerical Methods in Rock Mechanics" CivilZone Review paper which was published in Vol. 39, No. 4, June 2002, pp. 409–427, and is longer than usual papers in order to do justice to the subject. Also, an enhanced referencing system has been used here with the references being provided in two ways: firstly, by author and date, so that this information is contained directly in the text; and, secondly, by bracketted numbers, following the standard Journal format.

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1. Introduction

The purpose of this Journal Review Article is to present the techniques, advances, problems and likely future developments in numerical modelling for rock mechanics. In this Section, the review is prefaced by noting the special nature and idiosyncracies of rock masses—and hence some of the difficulties associated with capturing the rock reality in the numerical models. The utility of numerical modelling in providing understanding for rock engineering design and construction is also explained. Finally, we note here the scope and content of the Review with its emphasis on summarizing trends and providing an extensive literature source.

1.1. Special nature of rock masses

The reason for the general difficulty in modelling rock masses, by whatever numerical method, is that rock is a natural geological material, and so the physical or engineering properties have to be established, rather than defined through a manufacturing process. The rock mass is largely Discontinuous, Anisotropic, Inhomogeneous and Not-Elastic (DIANE), (Harrison and Hudson, 2000) [1]. Rock masses are under stress and continuously loaded by dynamic movements of the upper crust of the Earth, such as tectonic movements, earthquakes, land uplifting/subsidence, glaciation cycles and tides. A rock mass is also a fractured porous medium containing fluids in either liquid or gas phases, e.g. water, oil, natural gases and air, under complex in situ conditions of stresses, temperature and fluid pressures. The complex combination of constituents and its long history of formation make rock masses a difficult material for mathematical representation via numerical modelling.

In relation to the generally discontinuous nature of rock masses, the photograph of a blasted rock surface in Fig. 1 highlights the fact that rock masses contain through-going pre-existing fractures,[1] as well as fractures introduced by the excavation process.

Most of the fractures visible in Fig. 1 are pre-existing natural fractures. Although these rock fractures have occurred naturally through geological processes, their formation is governed by mechanical principles, as illustrated by the three main sets of fractures that, in this case, are mutually orthogonal and divide the rock mass into cuboids. The fractures are most often clustered in certain directions resulting from their geological modes and history of formation. One of the main tasks of numerical modelling in rock mechanics is to be able to characterize such mechanical discontinuities in a computer model—either explicitly or implicitly—the so-called 'material conceptualization'. Additionally, the interaction between the rock mass and the engineering structure has to be incorporated in the modelling procedure for design, so that consequences of the construction process have also to be characterized.

To adequately represent the rock mass in computational models, capturing such fracturing and the complete DIANE nature of the rock mass, plus the consequences of engineering, it is necessary to be able to include the following features during model conceptualization:

Fig. 1. Surface of a blasted rock mass, illustrating that pre-existing fractures can divide the rock mass into discrete blocks, and that the interaction between the rock mass and the engineering processes also needs to be modelled for the engineer to have a predictive capability for design purposes. Note the 'half-barrels' of the blasting boreholes.

- the relevant physical processes and their mathematical representations by partial differential equations (PDEs), especially when coupled thermal, hydraulic and mechanical processes need to be considered simultaneously;
- the relevant mechanisms and constitutive laws with the associated variables and parameters;
- the pre-existing state of rock stress (the rock mass being already under stress);
- the pre-existing state of temperature and water pressure (the rock mass is porous, fractured, and heated by a natural geothermal heat gradient or man-made heat sources)
- the presence of natural fractures (the rock mass is discontinuous);

[1] The word 'fracture' is used in this Article to indicate natural breaks in the rock continuum, e.g. faults, joints, bedding planes, fissures. Thus, the term 'fracture' is used here as a synonym for 'discontinuity'.

- variations in properties at different locations (the rock mass is inhomogeneous);
- variations of properties in different directions (the rock mass is anisotropic);
- time/rate-dependent behaviour (the rock mass is not elastic and may undergo creep or plastic deformation);
- variations of properties at different scales (the rock mass is scale-dependent);
- the effects resulting from the engineering perturbations (the geometry is altered).

The extent to which these features can actually be incorporated into a computer model will depend on the physical processes involved and the modelling technique used; hence, both the modelling and any subsequent rock engineering design will contain subjective judgements.

Rock engineering projects are becoming larger and more demanding in terms of the modelling requirements, one of which, for example, may be to include coupled thermo-hydro-mechanical (THM) behaviour into the model. A truly fully coupled model (including extra processes, such as chemistry) requires complete knowledge of the geometrical and physical properties and parameters of the fractured rock masses. Thus, the challenge is to know how to develop an adequate model. The model does not have to be complete and perfect: it only has to be adequate for the purpose.

For these reasons, rock mechanics modelling and rock engineering design are both a science and an art. They rest on a scientific foundation but require empirical judgements supported by accumulated experiences through long-term practices. This is the case because the quantity and quality of the supporting data for rock engineering design and analysis can never be complete, even though they can be perfectly defined in models.

1.2. Rock mechanics modelling for rock engineering design and construction

Some form of predictive capability is necessary in order to coherently design an engineered structure, whether it be on the rock mass surface or within the underground rock mass, and whether it be for civil engineering addressed in this CivilZone review or for mining, petroleum or environmental engineering. The predictive capability is achieved through a variety of modelling methods. Even if one simply adopts the same design as a previously constructed structure, the rock mass condition is generally site-specific and one should use a computer model adopted for the specific site conditions to ensure that the rock mass is likely to behave in similar fashion.

As rock mechanics modelling has developed for the design of rock engineering structures with widely different purposes, and because different modelling methods have been developed, we now have a wide spectrum of modelling approaches. These can be presented in different ways: the categorization into eight approaches based on four methods and two levels, as illustrated in Fig. 2, is from (Hudson, 2001) [2].

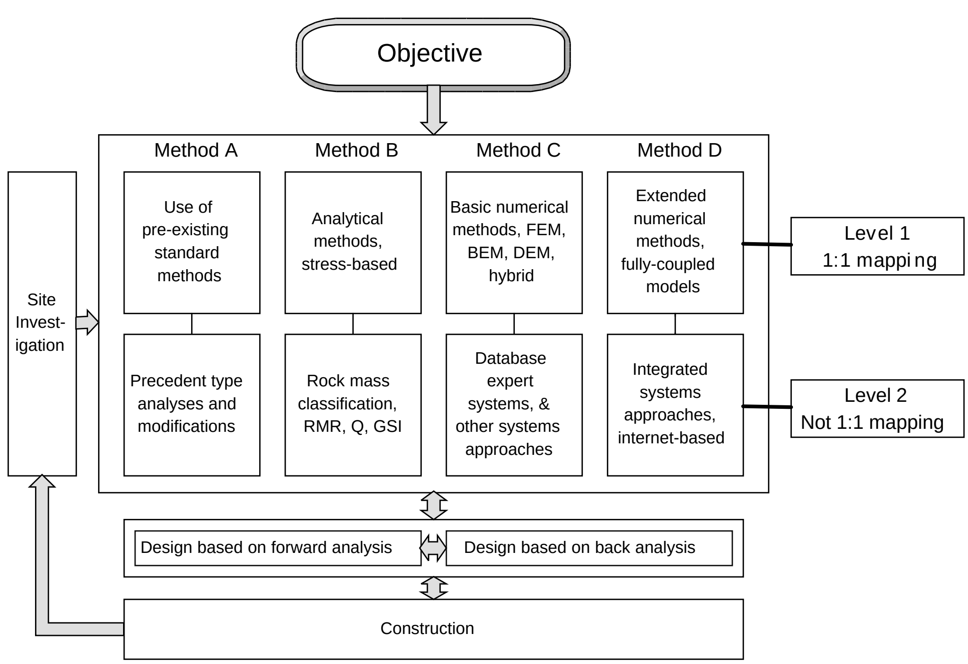

Fig. 2. Four basic methods, two levels and hence eight different approaches to rock mechanics modelling and rock engineering design, from Hudson [2].

The modelling and design work starts with the objective, the top box in Fig. 2. Then there are the eight modelling and design methods in the main central box. The four columns represent the four main modelling methods:

- Method A: Design based on previous design experiences,
- Method B: Design based on simplified models,
- Method C: Design based on modelling which attempts to capture most relevant mechanisms, and
- Method D: Design based on 'all-encompassing' modelling.

There are two rows in the large central box in Fig. 2. The top row, Level 1, includes methods in which there is an attempt to achieve one-to-one mechanism mapping in the model. In other words, a mechanism which is thought to be occurring in the rock reality and which is to be included in the model is modelled directly, such as explicit stress–strain relations. Conversely, the lower row, Level 2, includes methods in which such mechanism mapping is not direct. The consequences of, for example, the constitutive models and associated parameters may well be contained within the four modelling and design methods in Level 2, but one cannot explicitly identify the relation within the methodologies, e.g. in the rock mass classification techniques.

Some supporting rock mass characterization parameters will be obtained from site investigation, the left-hand box. Then the rock engineering design and construction proceeds, with a feedback loop to the modelling from construction.

An important point is that in rock mechanics and engineering design, having insufficient data is a way of life, rather than a simple local difficulty, and that is why the empirical approaches (i.e. classification systems) have been developed and are still required. Therefore, we will also be discussing the subject of parameter representability associated with sample size, representative elemental volume (REV), homogenization/upscaling, because these are fundamental problems associated with modelling, and are relevant to the ABCD method categories in Fig. 2.

1.3. Scope of this review

The use of computers makes significant contributions to all the eight modelling and design methods in Fig. 2; however, the specific numerical methods and approaches that are being reviewed here are used directly in Methods 1C and 1D. Also, there is concentration on the actual numerical methods (rather than computing per se or design per se) and discussion on the rock mass characterization issues related to the numerical methods. Highlighted are the techniques, advances, coupled mechanisms, technical auditing and the ability to present the content of the modelling, the outstanding issues, and the future of this type of modelling. In short, highlighted is the special contribution that numerical models are currently making to rock mechanics.

Because the focus of this Review is on the modelling concepts, the associated special features of modelling rock fractures, the main development milestones, typical application requirements, development trends, and outstanding issues of importance and difficulty, special attention is paid to Section 3 for alternative formulations in each of the modelling methods, noting the potentials for rock mechanics problems. It is hoped that this treatment will provide readers with a comprehensive presentation of the state-of-the-art of numerical analysis in rock mechanics in general, and civil engineering applications in particular—in terms of historical background, presents status and likely future trends.

2. Numerical methods in rock mechanics

Before considering the details and advances in the specific numerical modelling methods (presented in Section 3), an introduction is provided here to the methods and there is discussion on the continuum vs. discrete approaches. Also considered is the characterization of rock masses which is necessary to provide input to the numerical models, and there is illustration of how enhanced understanding is obtained through the use of such models.

2.1. Numerical methods for modelling continuous and discontinuous rock masses

In numerical modelling of engineering problems, some problems can be represented by an adequate model using a finite number of well-defined components. The behaviour of such components is either well known, or can be independently treated mathematically. The global behaviour of the system can be determined through well-defined inter-relations between the individual components (elements). One typical example of such discrete systems is a beam structure. Such problems are termed *discret*e and the discrete representation and solution of such systems by numerical methods are usually straightforward.

In other problems, the definition of such independent components may require an infinite sub-division of the problem domain, and the problem can only be treated using the mathematical assumption of an *infinitesimal element*, implying in theory an infinite number of components. This usually leads to differential equations to describe the system behaviour at the field points. Such systems are termed *continuous* and have infinite degrees of freedom. To solve such a continuous problem by numerical methods using digital computers, the problem domain is usually subdivided into a finite number of sub-domains (elements) whose behaviour is approximated by simpler mathematical descriptions with finite degrees of freedom. These sub-domains must satisfy both the governing differential equations of the problem and the continuity condition at their interfaces with adjacent elements. This is the so-called discretization of a continuum. It is an approximation of a continuous system with infinite degrees of freedom by a discrete system with finite degrees of freedom.

The continuity referred to here is a macroscopic concept. The continuum assumption implies that at all points in a problem domain, the materials cannot be torn open or broken into pieces. All material points originally in the neighbourhood of a certain point in the problem domain remain in the same neighbourhood throughout the deformation or transport process. Of course, at the microscopic scale, all materials are discrete systems. However, representing the microscopic components individually is intractable mathematically and unnecessary in practice.

The individual components (elements) of a discrete system are usually treated as continuous. Their properties may either be obtained from laboratory tests if the components are indeed continuous and macroscopically homogeneous, such as elastic beam structures, or be mathematically derived from homogenization processes if the components themselves are heterogeneous or/and fractured, such as the fractured rock masses we are considering here. The concepts of continuum and discontinuum are therefore not absolute but relative and problem-specific, depending especially on the problem scales. This is particularly true for rock mechanics problems. For example, a block of rock isolated by large fractures zones may be treated as one of many block components in a computer model, but the block itself may contain a large number of smaller fractures that cannot be explicitly represented if the problem is to be tractable. Homogenization is then needed to derive the equivalent continuum properties of such blocks, which are then functions of the geometry of the contained fracture systems and physical properties of the intact rock matrix and the fractures.

The fractured rock mass comprising the Earth's upper crust is a discrete system. Closed-form solutions do not exist for such geometries and numerical methods must be used for solving practical problems. Due to the differences in the underlying material assumptions, different numerical methods have been developed for continuous and discrete systems.

The most commonly applied numerical methods for rock mechanics problems are:

Continuum methods

- the Finite Difference Method (FDM),
- the Finite Element Method (FEM), and
- the Boundary Element Method (BEM).

Discontinuum methods

- Discrete Element Method (DEM),
- Discrete Fracture Network (DFN) methods.

Hybrid continuum/discontinuum models

- Hybrid FEM/BEM,
- Hybrid DEM/DEM,
- Hybrid FEM/DEM, and
- Other hybrid models.

The FDM is a direct approximation of the governing PDEs by replacing partial derivatives with differences at regular or irregular grids imposed over problem domains, thus transferring the original PDEs into a system of algebraic equations in terms of unknowns at grid points. The solution of the system equation is obtained after imposing the necessary initial and boundary conditions. This method is the oldest member in the family of numerical methods, one that is widely applied and is the basis of the explicit approach of the DEMs.

The FEM requires the division of the problem domain into a collection of sub-domains (elements) of smaller sizes and standard shapes (triangle, quadrilateral, tetrahedral, etc.) with fixed number of nodes at the vertices and/or on the sides—the discretization. Trial functions, usually polynomial, are used to approximate the behaviour of PDEs at the element level and generate the local algebraic equations representing the behaviour of the elements. The local elemental equations are then assembled, according to the topologic relations between the nodes and elements, into a global system of algebraic equations whose solution then produces the required information in the solution domain, after imposing the properly defined initial and boundary conditions. The FEM is perhaps the most widely applied numerical method in engineering today because its flexibility in handling material heterogeneity, non-linearity and boundary conditions, with many well developed and verified commercial codes with large capacities in terms of computing power, material complexity and user-friendliness. (It is also the basis of the implicit approach of the DEM.) Due to the interior discretization, the FDM and FEM cannot simulate infinitely large domains (as sometimes presented in rock engineering problems, such as half-plane or half-space problems) and the efficiency of the FDM and FEM will decrease with too high a number of degrees of freedom, which are in general proportional to the numbers of nodes.

The BEM, on the other hand, requires discretization at the boundary of the solution domains only, thus reducing the problem dimensions by one and greatly simplifying the input requirements. The information required in the solution domain is separately calculated from the information on the boundary, which is obtained by solution of a boundary integral equation, instead of direct solution of the PDEs, as in the FDM and FEM. It enjoys greater accuracy over the FDM and FEM at the same level of discretization and is also the most efficient technique for fracture propagation analysis. It is also best suited for simulating infinitely large domains due to the use of fundamental solutions of the PDEs in such domains.

The DEM for modelling a discontinuum is relatively new compared with the three methods described above and focuses mostly on applications in the fields of fractured or particulate geological media. The essence of the DEM is to represent the fractured medium as assemblages of blocks formed by connected fractures in the problem domain, and solve the equations of motion of these blocks through continuous detection and treatment of contacts between the blocks. The blocks can be rigid or be deformable with FDM or FEM discretizations. Large displacements caused by rigid body motion of individual blocks, including block rotation, fracture opening and complete detachments is straightforward in the DEM, but impossible in the FDM, FEM or BEM.

Fig. 3 illustrates the discretization concepts of the FDM/FEM, BEM and DEM for fractured rocks.

An alternative DEM for fluid flow in fractured rock masses is the DFN method that simulates fluid flow through connected fracture networks, with the matrix permeability either ignored or approximated by simple means. The stress and deformation of the fractures are generally ignored as well. This method is conceptually attractive for simulating fluid flow in fractured rocks when the permeability of the rock matrix is low compared to that of the fractures, and has wide applications in groundwater flow for civil engineering, reservoir simulation in petroleum engineering and heat energy extraction in geothermal engineering.

An important difference between the continuum and discrete methods is the treatment of displacement compatibility conditions. In the continuum methods, the displacement compatibility must be enforced between internal elements, which is automatic in the cases of the FDM and BEM, but for the FEM it is maintained by keeping constant element-node connectivity topology and consistent orders of the trial (shape) functions between the neighbouring elements. However, displacement compatibility is not required between blocks in the DEM, and is replaced by the contact conditions between blocks with specially developed constitutive models for point contacts or fractures.

The complete decoupling of rigid body motion mode and continuous deformation mode of individual blocks is usually adopted in DEM through the co-rotation scheme. The rigid body motion does not produce strains

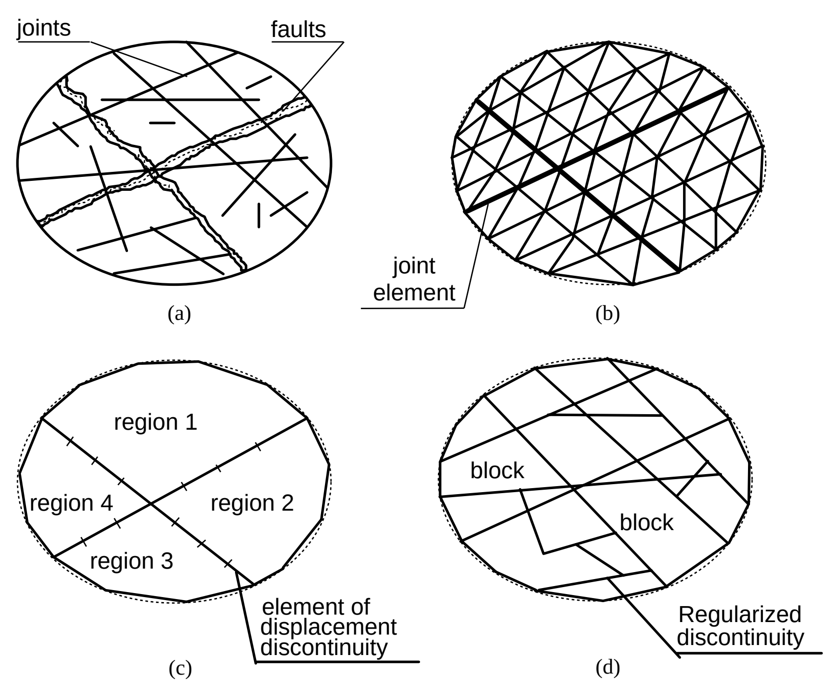

Fig. 3. Representation of a fractured rock mass shown in (a), by FDM or FEM shown in (b), BEM shown in (c), and DEM shown in (d).

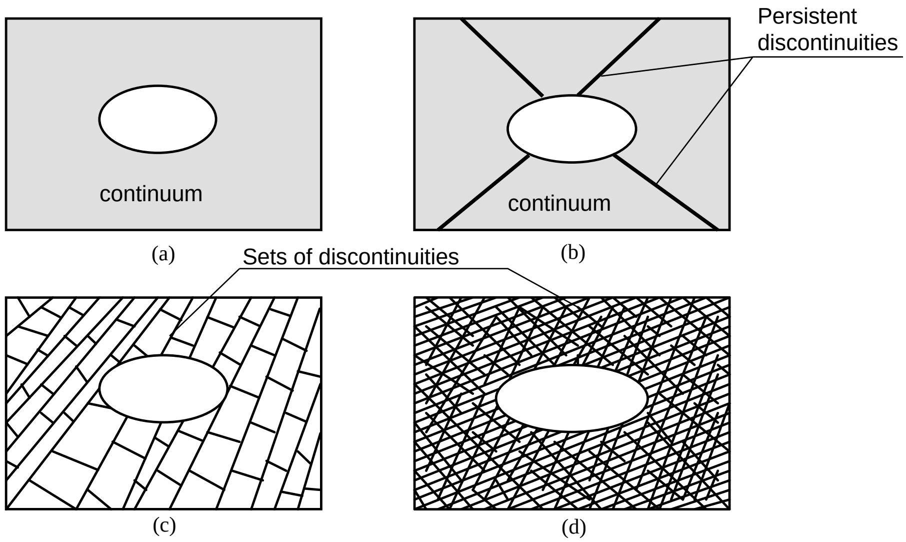

Fig. 4. Suitability of different numerical methods for an excavation in a rock mass: (a) continuum method; (b) either continuum with fracture elements or discrete method; (c) discrete method; and (d) continuum method with equivalent properties.

inside the blocks, but it does produce displacements of blocks, often of large scale. In the continuum approach, the rigid body motion mode of deformation is generally not included because it does not produce strains in the elements. Therefore, a continuous system reflects mainly the ''material deformation'' of the system and the discrete system reflects mainly the ''member (unit, or component) movement'' of the system.

The choice of continuum or discrete methods depends on many problem-specific factors, but mainly on the problem scale and fracture system geometry. Fig. 4 illustrates the alternative choices for different fracture circumstances in rock mechanics problems. Continuum approaches should be used for rock masses with no fractures or with many fractures, the behaviour of the latter being established through equivalent properties established by a homogenization process (Fig. 4a and d). The continuum approach can be used if only a few fractures are present and no fracture opening and no complete block detachment is possible (Fig. 4b). The discrete approach is most suitable for moderately fractured rock masses where the number of fractures too large for continuum-with-fracture-elements approach, or where large-scale displacements of individual blocks are possible (Fig. 4c).

Modelling fractured rocks demands high performance numerical methods and computer codes, especially regarding fracture representations, material heterogeneity and non-linearity, coupling with fluid flow and heat transfer and scale effects. It is often unnecessarily restrictive to use only one method, even less one code, to provide adequate representations for the most significant features and processes: hybrid models or multiple process codes are often used in combination in practice.

There are no absolute advantages of one method over another, as is explained further in the later part of this review. However, some of the disadvantages inherent in one type can be avoided by combined continuum-discrete models, termed hybrid models. In 1984, Lorig and Brady [3] presented an early computational scheme in which the far-field rock is modelled as a transversely isotropic continuum using the BEM and the near-field rock as a set of discrete element blocks defined by rock fractures. This type of hybrid BEM-DEM is shown in Fig. 5. The complex rock mass behaviour caused by fractures and matrix non-linearity in the near-field of the excavation can be efficiently handled by the DEM or FEM, surrounded by a BEM representation of the far-field region with linear material behaviour without fractures. The basis for such simple representation of the far-field is the fact that the gradients of variation of the physical variables, such as stress, displacement or flow, decrease rapidly with distance from the excavation. Therefore, if the interface between the near-field

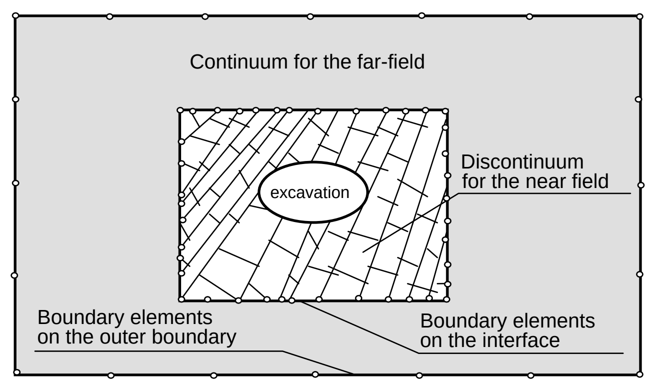

Fig. 5. Hybrid model for a rock mass containing an excavation—using the DEM for the near-field region close to the excavation and the BEM for the far-field region.

FEM/DEM region and far-field BEM region is far enough from the excavation, the BEM representation will provide an accurate enough representation to model the effects of the far-field on the near-field.

The 'model' and the 'computer' now provide essential support for rock mechanics analyses and understanding. The numerical methods and computing techniques assist in formulating conceptual models and mathematical developments to integrate and unify diverse geological, mechanical, hydraulic and thermal phenomena, whose interactions would not otherwise be revealed otherwise. Moreover, such developments and progress in computer methods for rock engineering will continue because they are mainly stimulated by the prospect that they will provide the information that cannot be obtained by experiments, because conducting large-scale in situ experiments is most often not possible.

In fact, full verification of computer models by experiments in rock mechanics is not possible: this is because the complete geometry and properties of the fractured rock mass components will never be completely known, and so verifications/validations can only be partial. Working with uncertainty and variability (about processes, properties, parameters, loading conditions and histories, initial and boundary conditions, etc.) becomes a way of life in rock engineering, requiring clarification of source information, understanding the significance of assumptions, studying propagation paths relating to the assumptions and their mathematical treatment.

Clearly defined mathematical approaches do exist to describe, analyse, and model uncertainties and error propagation, but their application in mathematical and computer models for rock engineering is still difficult—simply because we do not have a reference point for making judgements, except for broad empirical judgements. Modelling errors may be found as a result of the failure of rock engineering structures or accidents, but conceptual failures and modelling mistakes may be hidden under the thick blanket of the operational success of structures. Model reliability and credibility are always relative, subjective and case-dependent. This current lack of a rigorous treatment of uncertainty in rock engineering may well be a major reason why many practising engineers, and even researchers, remain uncertain about the validity and hence applicability of mathematical models and computer methods.

2.2. Characterization of rock masses for numerical methods

For the different types of numerical modelling methods described in Section 2.1, the modelling is linked to generic or specific rock masses by the boundary and initial conditions and the rock properties. For example, the elastic modelling of a tunnel excavation at a specific location requires a knowledge of the in situ rock stress state and the elastic properties of the rock. If the modelling is to incorporate the main components of the rock reality—the fractures, inhomogeneity, anisotropy and inelasticity, including failure—a more extensive model and a more extensive rock mass characterization are required. The scale effect is a related and additional problem, especially where fractures are affecting the rock mass properties (da Cunha, 1990, 1993; Amadei, 2000) [4,5,6].

Some of the rock characterization problems are as follows:

- the in situ rock stress is not easy to characterize over the region to be modelled;
- rock properties measured in the laboratory may not represent the values on a larger scale;
- rock properties cannot be measured directly on a large scale;
- rock properties may have to be estimated from empirical characterization techniques;
- the uncertainty in the rock property estimates is not easy to quantify.

These problems do not mean that we cannot supply the necessary rock characterization parameters but they do mean that the whole issue of rock characterization in relation to numerical methods must be carefully considered. Needless to say, the use of different numerical models will require different types of rock property characterization. Thus, the question of whether the numerical modelling is successful in capturing the rock reality relates to both the type of numerical model and the associated rock property characterization.

Other connected issues are:

- is it necessary to explicitly represent the fractures or can equivalent properties be used, i.e. discontinuum vs. continuum models?
- to what extent is it necessary to simulate all the operating mechanisms, i.e. to use a 1:1 mechanism mapping approach, cf. Level 1 vs. Level 2 in Fig. 2?
- how can the combined numerical modelling technique and rock characterization method be calibrated?
- how can the rock characterization method be technically audited to provide some guidance on whether it is an adequate procedure?

These issues are discussed in Section 5.

2.3. Enhanced understanding provided by numerical methods

The purpose of numerical modelling in rock mechanics is not only to provide specific values of, say, stress and strain, at specific points but is also to enhance

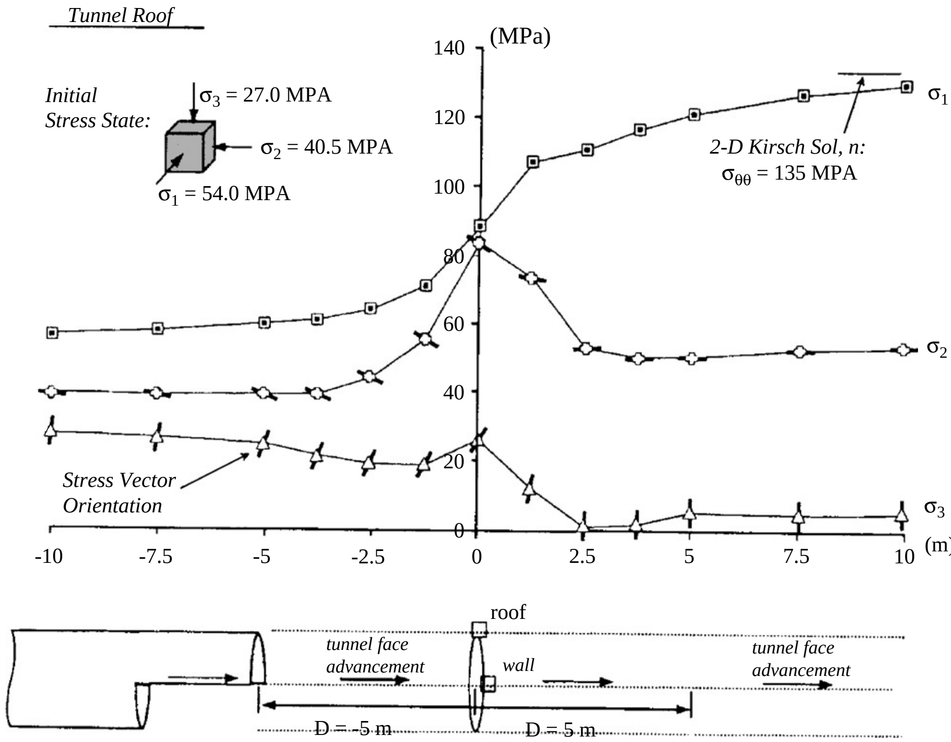

Fig. 6. Variation in the 3-D stress state along the roof line of an advancing tunnel face, from Eberhardt (2001) [7].

our understanding of the processes involved, particularly the changes that result from the perturbations introduced by the engineering. For example, we can use different numerical models based on linear elasticity to estimate the rock stress changes that occurs as a result of tunnelling. We can answer the question 'What is the maximal compressive stress in the wall of a tunnel after excavation?' However, enhanced understanding comes from a numerical demonstration of the progressive change in stresses throughout the full stress path from the original natural rock stress state to the final disturbed stress state. The illustration in Fig. 6 shows the variation in the three-dimensional (3-D) stress state ahead of an advancing tunnel face and provides much more information than just the before and after values Eberhardt (2001) [7]. Also, the effects of engineering actions, such as the time when support is introduced, can be studied more coherently.

Similarly, enhanced understanding comes from studying the development of all the processes that occur in rock masses as a result of engineering actions. Some of the advantages of numerical modelling in this context are the ability to:

- study rock mechanics processes from beginning to end,
- conduct sensitivity analyses rapidly,
- conduct numerical experiments for design and construction options,

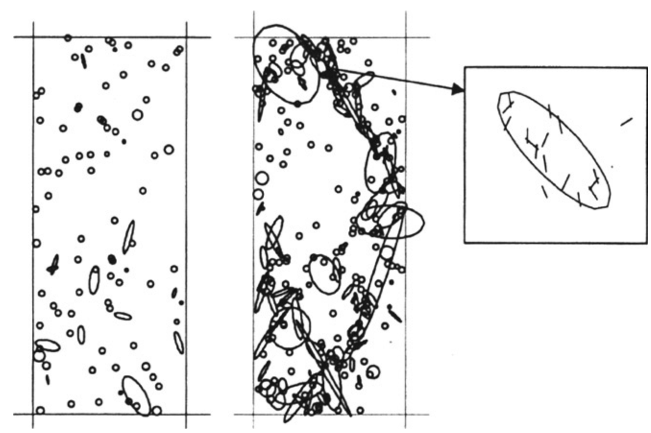

Fig. 7. Use of the Particle Flow Code (PFC) to model progressive failure and acoustic emission in the uniaxial compression test (from Hazzard and Young, 2001 [8]).

- develop qualitative understanding through quantitative evaluation,
- establish to what extent 1:1 mapping of mechanisms and properties is necessary.

An example of studying the development of a fundamental failure mechanism with a numerical code is given in Fig. 7 where the development of fracturing during the uniaxial compression of a rock specimen is shown (Hazzard and Young, 2000, [8]). This is a good example of a numerical model being able to illustrate the progressive failure mechanism. This approach can be applied to modelling the rock mass response to engineering actions in different field circumstances, leading to enhanced understanding and hence enabling the engineer to design more coherently.

3. Numerical techniques for rock mechanics: states-of-the-art

3.1. Finite Difference Methods

3.1.1. Basic concepts

The FDM is the oldest numerical method to obtain approximate solutions to PDEs in engineering, especially in fluid dynamics, heat transfer and solid mechanics. The basic concept of FDM is to replace the partial derivatives of the objective function (e.g. displacement) by differences defined over certain spatial intervals in the coordinate directions, Δx, Δy, Δz, which yields a system of algebraic simultaneous equations of the objective functions at a grid (mesh) of nodes over the domain of interest (Fig. 8a) (Wheel, 1996 [9]). Solution of the simultaneous algebraic system equations, incorporating boundary conditions defined at boundary nodes, will then produce the required values of the objective function at all nodes, which satisfy both the governing PDFs and specified boundary conditions. The conventional FDM utilizes a regular grid of nodes, such as a rectangular grid as shown in Fig. 8a.

Using a standard FDM scheme, the so-called 5-point difference scheme (Fig. 8b), the resultant FDM equation at grid node (i,j) will be expressed as combinations of function values at its four surrounding nodes. For a Navier equation of equilibrium for elastic solids in 2-D, the FDM equation of equilibrium at point (i,j) is given as

$$u_x^{i,j} = a_1 u_x^{i-1,j} + a_2 u_x^{i,j-1} + a_3 u_x^{i,j+1} + a_4 u_x^{i+1,j} + a_5 u_x^{i+1,j+1} + a_6 F_x^{i,j},$$

$$u_y^{i,j} = b_1 u_y^{i-1,j} + b_2 u_y^{i,j-1} + b_3 u_y^{i,j+1} + b_4 u_y^{i+1,j} + b_5 u_y^{i+1,j+1} + b_6 F_y^{i,j}, \quad (1)$$

where coefficients a_k and $b_k (k = 1, 2, \ldots, 6)$ are functions of the grid intervals Δx and Δy, and the elastic properties of the solids, and $F_x^{i,j}$ and $F_y^{i,j}$ are the body forces lumped at point (i,j), respectively. Assembly of similar equations at all grid points will yield a global system of algebraic equations whose solution can be obtained by direct or iterative methods. FDM schemes can also be applied in the time domain with properly chosen time steps, Δt, so that function values at time t can be inferred from values at $t - \Delta t$.

The fundamental nature of FDM is the direct discretization of the governing PDEs by replacing the partial derivatives with differences defined at neighbouring grid points. The grid system is only a convenient way of generating objective function values at sampling points with small enough intervals between them, so that errors thus introduced are small enough to be acceptable. No local trial (or interpolation) functions are employed to approximate the PDE in the neighbourhoods of the sampling points, as is done in FEM and BEM. It is therefore the most direct and intuitive technique for the solution of the PDEs. The conventional FDM with regular grid systems does suffer from shortcomings, most of all in its inflexibility in dealing with fractures, complex boundary conditions and material inhomogeneity. This makes the standard FDM generally unsuitable for modelling practical rock mechanics problems. However, significant progress has been made in the FDM so that irregular meshes, such as quadrilateral grids (Perrone and Kao, 1975, [10]) and the Voronoi grids (Brighi et al., 1998, [11]) can also be used. Although such irregular meshes can enhance the applicability of the FDM for rock mechanics problems, however, the most significant improvement comes from the so-called Control Volume or Finite Volume approaches.

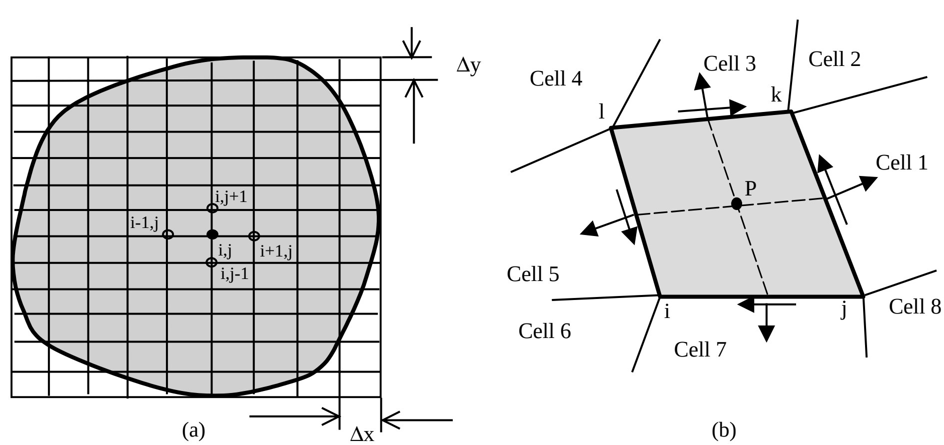

Fig. 8. (a) Regular quadrilateral grid for the FDM and (b) irregular quadrilateral grid for the FVM (after Wheel, 1995 [9]).

3.1.2. Finite volume approach of FDM and its application to stress analysis

The Finite Volume Method (FVM) is also a direct approximation of the PDEs, but in an integral sense. An elastostatics problem with body Ω, is divided into a finite number, N, of internal contiguous cells of arbitrary polyhedral (or polygonal in 2-D cases) shape, called Control Volumes (CV) Ω_k with boundary Γ_k of unit outward normal vector n_i^k, $k = 1, 2, \ldots, N$. The boundary Γ_k of CV Ω_k is comprised of a number, M^k, polygonal side (faces or line segments), Γ_k^p, $p = 1, 2, \ldots, M^k$, $\Gamma_k = \cup_{p=1}^{M^k} \Gamma_k^p$. Assuming isotropic, linear elasticity and using Gauss' divergence theorem, the Navier–Cauchy equation of equilibrium in terms of stress can be rewritten in terms of displacement as

$$\sum_{k=1}^{N}\left[\sum_{p=1}^{M^k}\int_{\Gamma_k^p} t_i^k \,\mathrm{d}\Gamma + \int_{\Omega_k} f_i \,\mathrm{d}\Omega\right] = \sum_{k=1}^{N}\left[\sum_{p=1}^{M^k}\int_{\Gamma_k^p} \sigma_{ij}^k n_j^p \,\mathrm{d}\Gamma + F_x^k\right] = 0, \tag{2}$$

where $F_i^k = \rho g_i V^k$ is the body force vector of the CV of volume V^k lumped at its centre, ρ is the material density and g_i is the body force intensity vector, such as gravity acceleration.

The task is to formulate the integrals into algebraic functions of the displacements at nodes defining the boundary sides Γ_k^p of Ω_k, which vary with different grid schemes. For an unstructured quadrilateral grid system (Fig. 8b), a typical cell P (CV), with its centre at node P, has four sides (ij, jk, kl, li) and four nodes (i, j, k, l), surrounded by eight neighbouring cells with centre nodes $I, J, \ldots, O$. The integral terms in Eq. (2) for the cell P are written in terms of displacement variables at the centres of cells [9], written as

$$\begin{aligned} A_p u_x^p + \sum_r A_r u_x^r + B_p u_y^p + \sum_r B_r u_y^r + F_x^K &= 0, \\ C_p u_y^p + \sum_r C_r u_y^r + D_p u_x^p + \sum_r D_r u_x^r + F_y^K &= 0, \end{aligned} \tag{3}$$

where coefficients $A_p, A_r, B_p, B_r, C_p, C_r, D_p, D_r$ are functions of the cell geometry and the elastic properties of the solids, with $r = 1, 2, \ldots, 8$ running through the eight surrounding cells.

This formulation of FVM with displacement variables at cell centres is called the cell-centred scheme of the FVM. If, on the other hand, the nodal displacement variables are kept as the system unknowns and the displacements at the cell centres are replaced by a combination of nodal displacements defining the cells, the scheme is called the vertex-centred scheme of the FVM. It is also possible to consider different material properties in different cells in the FVM, in similar ways as in the FEM. The FDM/FVM approach is therefore as flexible as FEM in handling material inhomogeneity and mesh generation.

As a branch of the FDM, the FVM can overcome the inflexibility of the grid generation and boundary conditions in the traditional FDM with unstructured grids of arbitrary shape. It has similarities with the FEM and is also regarded as a bridge between FDM and FEM, as pointed out in Selim (1993) and Fallah et al. (2000) [12,13]. A FVM model can be readily constructed using a standard FEM mesh, as shown in Bailey and Cross (1995) [14]. Similar examples of FVM for non-linear stress analysis with elasto-plastic and visco-plastic material models is given in Fryer et al. (1991) [15].

With proper formulations, such as static or dynamic relaxation techniques, no global system of equations in matrix form needs to be formed and solved in the FDM/FVM approach. The formation and solution of the equations are localized, which is more efficient for memory and storage handling in the computer implementation. This also provides the additional advantage of more straightforward simulation of complex constitutive material behaviour, such as plasticity and damage, without iterative solutions of predictor–corrector mapping schemes that must be used in other numerical methods using global matrix equation systems, as in the FEM or BEM. The FDM/FVM approaches are therefore specially suited to simulate non-linear behaviour of solid materials. The reason is its special advantage of no-matrix-equation-solving formulation and data structure, so that integration of non-linear constitutive equations is a straightforward computer implementation step, rather than iterative prediction-mapping integration loops required in FEM. This is one of the main attractiveness of FVM such as demonstrated in Winkins (1963) and Taylor et al. (1995) [16,17]. At present, the most well-known computer codes for stress analysis for non-linear rock engineering problems using the FVM/FDM approach is perhaps the FLAC code group (ITASCA, 1993) [18], with a vertex scheme of triangle and/or quadrilateral grids.

3.1.3. Fracture and non-linear analyses with FDM/FVM

Explicit representation of fractures is not easy in FDM/FVM because the finite difference schemes in FDM and interpolations in FVM require continuity of the functions between the neighbouring grid points. During the early development of FVM approaches, it is possible to represent weakness zones of certain thickness as collections of cells of different materials, which are not permitted to have openings or to be detached from their neighbouring cells. However, it is possible today to have special "fracture elements" in FVM models as in FEM, such as reported in Granet et al. (2001) and Caillabet et al. (2000) [19,20] for fluid flow in deformable porous media. On the other hand, the FDM/FVM models have been used to study the mechanisms of macroscopic fracturing processes, such as shear-band formation in the laboratory testing of rock and soil samples (Fang, 2000; Martino et al., 2002) [21,22], slope stability (Kourdey et al., 2001) [23] and glacial dynamics (Marmo and Wilson, 2001) [24]. This is achieved as a process of material failure or damage propagation at the grid points or cell centres, without creating fracture surfaces in the models.

Another important improvement of FVM over the classical FDM is the use of unstructured meshes, such as triangles, arbitrary quadrilaterals, or Vorinoi grids (Mishev, 1998) [25]. This advantage, plus the flexibility of the FVM approach in material models and boundary condition enforcement, ensures that the FDM/FVM is still one of the most popular numerical methods in rock engineering, with applications covering almost all aspects of rock mechanics, e.g. slope stability, underground openings, coupled hydro-mechanical or THM processes, rock mass characterization, tectonic process, and glacial dynamics. The most comprehensive coverage in this regard can be seen in Detournay and Hart (1999) [26]. The late developments in fundamentals and computer formulations can be seen in Benito et al. (2001), Oñate et al. (1994), Lahrmann (1992), Demirdžić and Muzaferija (1994) Demirdžić et al. (2000), Jasak and Weller (2000) and Cocchi (2000) [27–33].

3.2. Finite Element Method and related methods

3.2.1. Basic concepts

Although the concept of domain discretization can be traced back to Courant (1943), and Prager and Synge (1947) [34,35], the ground-breaking work in FEM development is described in Turner et al. (1956) [36] when triangle elements were first invented for structural analysis (Clough, 1960, [37]) when the term FEM was first used for plane stress problems, and in Argyris (1960) [38] presenting the matrix method for structural analysis, and describing the duality of force and displacement transformations and the virtual work principle. The method was rapidly adopted and promoted in many scientific and engineering fields, as illustrated by the text books of Zienkiewicz (1977) and Bathe (1982) [39,40].

Indeed, the FEM has been the most popular numerical method in engineering sciences, including rock mechanics and rock engineering. Its popularity is largely due to its flexibility in handling material inhomogeneity and anisotropy, complex boundary conditions and dynamic problems, together with moderate efficiency in dealing with complex constitutive models and fractures, i.e. the DIANE features. All these merits were very appealing to researchers and practising engineers alike during early development in the 1960s and 1970s when the main numerical method in engineering analysis was the FDM with regular grids. Since then, the FEM method has been extended in many directions.

Basically, three steps are required to complete an FEM analysis: domain discretization, local approximation, and assemblage and solution of the global matrix equation. The domain discretization involves dividing the domain into a finite number of internal contiguous elements of regular shapes defined by a fixed number of nodes (e.g., triangle elements with three nodes in 2-D and brick elements with eight nodes in 3-D). A basic assumption in the FEM is that the unknown function, u_i^e over each element, can be approximated through a trial function of its nodal values of the system unknowns, u_i^j, in a polynomial form. The trial function must satisfy the governing PDF and is given by

$$u_i^e = \sum_{j=1}^{M} N_{ij} u_i^j, \tag{4}$$

where the N_{ij} are often called the shape functions (or interpolation functions) defined in intrinsic coordinates in order to use Gaussian quadrature integration, and M is the order of the elements. Using the shape functions, the original PDF of the problem is replaced by an algebraic system of equations written

$$\sum_{i=1}^{N} [K_{ij}^e]\{u_j^e\} = \sum_{i=1}^{N} (f_i^e) \quad \text{or} \quad \mathbf{Ku} = \mathbf{F}, \tag{5}$$

where matrix $[K_{ij}^e]$ is the coefficient matrix, vector $\{u_j^e\}$ is the nodal value vector of the unknown variables, and vector $\{f_i^e\}$ is comprised of contributions from body force terms and initial/boundary conditions.

For elasticity problems, the matrix $[K_{ij}^e]$ is called the element stiffness matrix given by

$$[K_{ij}^e] = \int_{\Omega_i} ([B_i][N_i])^{\mathrm{T}} [D_i][B_j]\, \mathrm{d}\Omega, \tag{6}$$

where matrix $[D_i]$ is the elasticity matrix and matrix $[B_i]$ is the geometry matrix determined by the relation between the displacement and strain. The global stiffness matrix $\mathbf{K}$ is banded and symmetric because the matrices $[D_i]$ are symmetric. Material inhomogeneity in FEM is most straightforwardly incorporated by assigning different material properties to different elements (or regions). To enforce the displacement compatibility condition, the order of shape functions along a common edge shared by two elements must be the same, so that no displacement discontinuity occurs along and across the edge.

"Infinite elements" have also been developed in FEM to consider the effects of an infinite far-field domain on the near-field behaviour, most notably the "infinite domain elements" of Beer and Meck (1981) [41] and the "mapped infinite elements" of Zienkiewicz et al. (1983) [42], with focus on geo-mechanical applications. The original concept was proposed by Bettess (1977) [43] for fluid mechanics problems. An infinite element formulation with body force terms was given recently by Cheng (1996) [44] with the emphasis also on geotechnical problems. The mapped infinite elements are simply implemented using special shape functions that project boundary nodes at infinite distances in one or two directions, where the displacements are either zero or have prescribed values. Additional nodes are needed at the imaginary infinite locations. The infinite domain element technique does not require additional infinite nodes, but requires a "decay function" to describe the manner in which the displacements vary from mesh boundary to infinity. The shape functions used in the infinite element formulations are singular at the "infinite" nodes.

Because rock mechanics is one of the most stimulating fields for development of numerical methods—with many special challenges, such as fractures, property heterogeneity and anisotropy, material and geometrical non-linearity, and scale and time effects—much FEM development work and application has been specifically oriented towards rock mechanics problems, as illustrated in the publications of Owen and Hinton (1980), Naylor et al. (1981), Pande et al. (1990), Wittke (1990), and Beer and Watson (1992) [45–49]. The FEM has been the most widely applied numerical methods for rock mechanics problems in civil engineering because it was the first numerical method with enough flexibility for treatment of material heterogeneity, non-linear deformability (mainly plasticity), complex boundary conditions, in situ stresses and gravity. A typical recent development is given in Tang et al. (1998) [50] for simulating fracturing processes in inhomogeneous rocks with FEM. Also, the method appeared in the late 1960s and early 1970s, when the traditional FDM with regular grids could not satisfy these essential requirements for rock mechanics problems. It out-performed the conventional FDM because of these advantages.

3.2.2. Fracture analysis with the FEM

Representation of rock fractures in the FEM has been motivated by rock mechanics needs since the late 1960s, with the most notably contributions from Goodman et al. (1968), Goodman (1976), Zienkiewicz et al. (1970), Ghaboussi et al. (1973), Katona (1983), Desai et al. (1984) [51–56].

Assuming that the contact stresses and relative displacements along and across the rock fractures of a theoretical zero thickness (Fig. 9a) follow a linear relation with constant normal and shear stiffness, K_n and K_s, Goodman et al. (1968) [51] proposed a 'joint element' which can be readily incorporated into an FEM process, with its local equilibrium equation given by

$$\mathbf{k}^G\mathbf{u}^G = \mathbf{f}^G, \tag{7}$$

where the matrix $\mathbf{k}^G$ is a symmetric matrix with its entries defined by the normal and shear stiffness, the element's length and its orientation to the global coordinate system, respectively. The vector $\mathbf{u}^G = (u_x^i, u_y^i, u_x^j, u_y^j, u_x^k, u_y^k, u_x^l, u_y^l)^T$ is the nodal displacement vector of the four nodes (i, j, k and l) defining the joint element (Fig. 9b) and vector $\mathbf{f}^G$.

The above formulation, the well-known 'Goodman joint element' in rock mechanics literature, has been widely implemented in FEM codes and applied to many practical rock engineering problems. Also, it has been extended to consider peak and post-peak behaviour in the shear direction. However, its formulation is based on continuum assumptions—so that large-scale opening, sliding, and complete detachment of elements are not permitted. The displacements of a joint element are of the same order of magnitude as its neighbouring continuum elements, allowing the displacement compatibility condition to be kept along and across the joint elements. Because of the zero thickness of the joint element, numerical ill-conditioning may arise due to large aspect ratios (the ratio of length to thickness) of joint elements.

Zienkiewicz et al. (1970) [53] proposed a six-node fracture element with two additional nodes in the

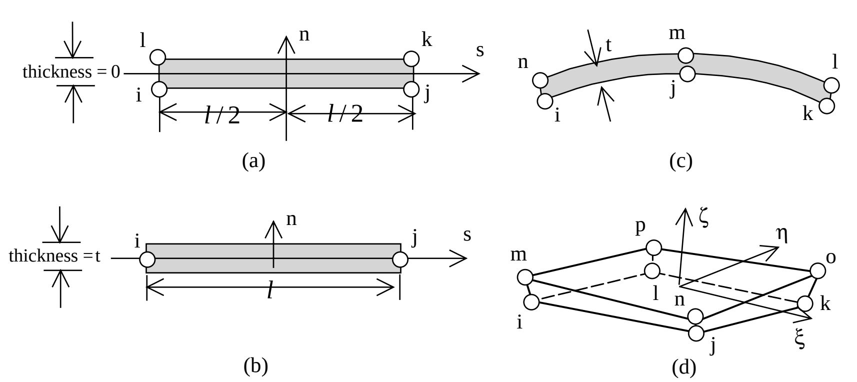

Fig. 9. Fracture elements in FEM by (a) Goodman et al. (1968) [51], (b) Ghaboussi et al. (1973) [54], (c) Zienkiewicz et al. (1970) [53] and (d) Buczkowski and Kleiber (1997) [60].

middle section of the element, and a small thickness (Fig. 9c). The elements can, therefore, be curved. The formulation may be seen as a 'degenerate' ordinary solid element of narrow thickness, and is subject to numerical ill-conditioning when the aspect ratio is too large.

Using the relative displacements between the two opposite surfaces of fractures as the independent system unknowns, Ghaboussi et al. (1973) [54] proposed an FEM joint element based on the theory of plasticity (Fig. 9b). The use of the relative displacement components across and along the fractures of finite thickness reduces the number of unknowns of the fracture elements by half, defined at two nodes instead of four nodes as in Goodman's joint elements. A finite thickness t is also used. The normal and shear strain components of the element are defined as the corresponding ratios of relative normal and shear displacements over the fracture thickness. An elasto-plastic relation between the normal and shear stresses and the normal and shear strains of the fracture element is formulated and can be implemented in the usual manner for continuum FEM analysis. This formulation is more robust in terms of numerical ill-conditioning as compared with those proposed in Goodman et al. (1968) and Zienkienwicz et al. (1970) [51,53], due to the use of the relative displacements.

The 'thin-layer' elements developed by Desai et al. (1984) [56] are also based on a continuum assumption; these are a solid element with a specially developed constitutive model for contact and frictional sliding.

The fracture element formulation in FEM has also been developed with interface element models in contact mechanics, using the FEM approach, instead of the continuum solid element approximation as mentioned above. Katona (1983) [55] developed an FEM interface element model defined by mating pairs of nodes, without using the normal and shear stiffness parameters, and with three states—sticking, slipping and opening—based on the Coulomb friction law. A similar approach was further discussed in Wang and Yuan (1997) [57]. In Gens et al. (1989, 1995) [58,59] 3-D FEM interface models simulating the behaviour of rock fractures were developed using the theory of plasticity. Based on the same principles, recent work by Buczkowski and Kleiber (1997) [60] considered orthotropic friction for contact interface elements in the FEM based on the theory of plasticity.

The FEM interface models described above present significant improvements over the early joint element models through a more systematic consideration of the kinetic and thermodynamic constraints, but they are still limited to the small displacement assumptions in the FEM with the consequence that large-scale movements across and along fracture elements are not possible. Despite these efforts, the treatment of fractures and fracture growth remains the most important limiting factor in the application of the FEM for rock mechanics problems, especially when large number of fractures needs to be represented explicitly. The FEM suffers from the fact that the global stiffness matrix tends to be ill-conditioned when many fracture elements are incorporated. Block rotations, complete detachment and large-scale fracture opening cannot be treated because the general continuum assumption in FEM formulations requires that fracture elements cannot be torn apart.

When simulating the process of fracture growth, the FEM is handicapped by the requirement of small element size, continuous re-meshing with fracture growth, and conformable fracture path and element edges. This overall shortcoming makes the FEM less efficient in dealing with fracture problems than its BEM counterparts.

However, special algorithms have been developed in an attempt to overcome this disadvantage, e.g. using discontinuous shape functions (Wan, 1990) [61] for implicit simulation of fracture initiation and growth through bifurcation theory.

A special class of FEM, often called 'enriched FEM', has been especially developed for fracture analysis with minimal or no re-meshing, as reported in Belytschko and Black (1999), Belytschko et al. (2001), Daux et al. (2000), Duarte et al. (2000, 2001), Dolbow et al. (2000), Jirasek and Zimmermann (2001a, b), Moës et al. (1999) and Sukumar et al. (2000) [62–71]. The basic concept is direct representation of the objective function (such as displacements) with arbitrary discontinuities and discontinuous derivatives in FEM, but without need for the FEM meshes to conform to the fractures and no need for re-meshing for fracture growth.

The treatment of fractures is at the element level. The surfaces of the fractures are defined by assigned distance functions so that their representation requires only nodal function values, represented by an additional degree of freedom in the trial functions, a jump function along the fracture and a crack tip function at the tips. The motions of the fractures are simulated using the level sets technique (Stolarska et al., 2001) [72]. Fig. 10a illustrates the non-conformal fracture-mesh relation in this technique with an arbitrary fracture intersecting a regular mesh, where circled nodes are 'enriched' with additional jump functions and squared nodes are 'enriched' with additional crack tip functions. On the other hand, the regular elements intersected by the fractures are changed into general polygons (Fig. 10b) and quadrature of the weak form in elements requires that these polygons be subdivided into standard FEM elements (such as triangles, Fig. 10c for numerical integration (Moës et al., 1999; Belytschko et al., 2001) [70,71]. In Belytschko et al. (2001) [63], the enriched

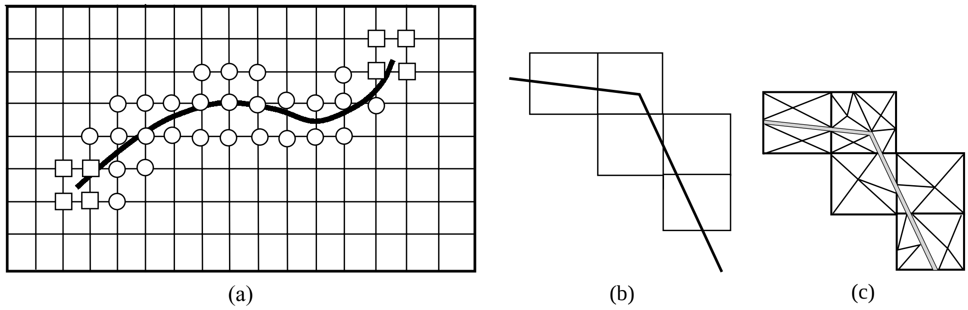

Fig. 10. (a) Representation of an arbitrary fracture in a regular FEM mesh with nodes enriched by jump functions (circled nodes) or crack tip functions (squared nodes) (after Moës et al., 1999) [70]; (b) Details of rectangular elements intersected by a fracture, thus forming polygons; and (c) triangularization of polygons into triangle elements for quadrature integration (after Belytschko et al., 2001) [63].

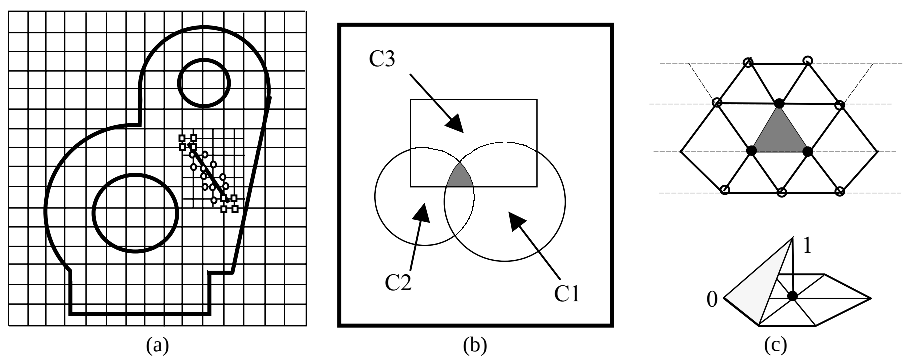

Fig. 11. (a) A regular non-conformal mesh of GFEM with 'enriched' nodes surrounding a fracture; (b) general coverings in the manifold method; and (c) manifold method coverings with a standard triangular FEM mesh and shape functions (after Chen et al., 1998 [78]).

method was applied to a tunnel stability analysis with fractures simulated as displacement discontinuities.

The 'enriched' FEM with jump functions and crack tip functions has improved the FEM's capacity in fracture analysis. Coupled with the FEM's advantage in dealing with material heterogeneity and non-linearity, this makes the 'enriched' FEM suitable for non-linear fracture analysis. One such example is the so-called 'generalized finite element method' (GFEM), which was developed based on the partition of unity principle (Duarte et al., 2000; Strouboulis et al., 2000, 2001) [73–75]. The mesh in GFEM is independent of the geometry of the domain of interest and therefore can be regular regardless of the object geometry. Fractures can be simulated by their surrounding nodes 'enriched' by jump functions and crack tip functions (Fig. 11a). This greatly simplifies the meshing tasks.

The GFEM is in many ways similar to the so-called 'manifold' method except for the treatment of fractures and discrete blocks (Shi, 1991, 1992; Chen et al., 1998) [76–78]. The manifold method uses the truncated discontinuous shape functions to simulate the fractures and treat the continuum bodies, fractured bodies and assemblage of discrete blocks in a unified form, and is a natural bridge between the continuum and discrete representations.

The method is formulated using a node-star covering system for constructing the trial functions (Fig. 11b). A node is associated with a covering—star, which can be a standard FEM mesh (Fig. 11c) or generated using least-square kernel techniques with general shapes. The integration, however, is performed analytically using Simplex integration techniques.

Like GFEM, the manifold method can also have meshes independent of the domain geometry, and therefore the meshing task is greatly simplified and simulation of the fracturing process does not need re-meshing. The technique has been extended for applications to rock mechanics problems with large deformations and crack propagation (Wang et al., 19971a, 1997b) [79,80]. Most of the publications are included in the series of proceedings of the ICADD[2] symposia (Li et al., 1995; Salami and Banks, 1996; Ohnishi, 1997; Amadei, 1999) [81–84].

3.2.3. Meshless (meshfree) methods

Besides the fracture analysis problem, the traditional FEM suffers from other shortcomings, especially the 'locking' effects and high demand for mesh generation.

[2] ICADD (acronym for International Conference on Analysis of Discontinuous Deformation).

There are two types of 'locking' effects in FEM: numerical locking and element locking. 'Numerical locking' is the phenomenon by which numerical approximation deteriorates near some limiting values of material properties (Arnold, 1981; Babuška and Suri, 1992; Suri, 1996) [85–87], or special geometry limits. Typical examples are the Poisson's ratio for elasticity problems when the value of Poisson's ratio is near 0.5, and shear locking for shells and plates when the thickness of the shells and plates is reduced to a small amount (Bucalen and Bathe, 1995) [88]. Proper h-, p- or hp-convergence measures need to be taken to avoid such locking effects and ensure solution convergence (Babuška and Suri, 1990; Chilton and Suri, 1997) [89,90].

The 'element locking' in FEM is the numerical instability and can be caused by local mesh distortions, such as large aspect ratios of elements under highly concentrated loads, especially in dynamic large deformation analysis. The mesh generation is a demanding task in applying FEM for practical problems with complex interior structures and exterior boundaries. The meshes must be detailed enough to ensure proper representation of the problem geometry, solution convergence and result accuracy; yet they should also enable the computations to be completed in an economically reasonable time. These conflicting issues are problem-specific and must be balanced carefully between resolution and resources. The problem is critical when dealing with 3-D problems with complex geometry.

Although commercial software is available for fully or semi-automatic generation of FEM meshes (as pre-processor) and for results evaluation and presentation (as post-processor), it is still up to the engineers and analysts as code users to address the issues of mesh resolution, result accuracy and computing resources. Mesh generation usually takes a much longer time than the actual calculations—because it depends almost entirely on judgement and experience, rather than theoretical guidance. A number of trial-and-error cycles are often needed to settle the issues properly.

The most common method for improving the solution convergence and avoiding locking effects is by successively increasing mesh resolution, i.e. increasing the number of elements. This is called the h-convergence approach. A different approach, called p-convergence, is to increase the order of the trial (shape) functions, i.e. increasing node numbers per element while maintaining a constant element numbers. The combination of the two approaches is the hp-convergence approach in FEM, and is built into many commercial FEM codes. The aim is to avoid the 'hourglass' phenomenon due mainly to too low an order of trial functions, and achieve a robust solution convergence rate (Oden, 1990; Babuška and Suri, 1990) [91,89].

Significant progress has been made in the last decade in the 'meshless' (or 'meshfree', 'element-free') method, which is closely related to FEM. In this approach, the trial functions are no longer standard but generated from neighbouring nodes within a domain of influence by different approximations, such as the least-square technique (Belytschko et al., 1996) [92]. The requirement for mesh generation is only generation and distribution of discrete nodes, without fixed element-node topological relations as in the FEM. A large number of different meshless formulations have been developed over the years. Most notable among them are:

- "Smooth particle hydrodynamics" method (SPH) (Monaghna, 1988; Randles and Libersky, 1996) [93,94];
- "Diffuse element method" by Nayroles et al. (1992) [95];
- "Element-free Galerkin method" (EFG) (Belytschko et al., 1994) [96];
- "Reproducing kernel particle methods" (RKPM) (Liu et al., 1995, 1996; Chen et al., 1996) [97–99];
- "Moving least squares reproducing kernel method" (MLSRK) (Liu et al., 1997) [100];
- "hp-cloud method" (Duarte and Oden, 1996; Liszka et al., 1996) [101,102];
- "Partition of unity method" (PUM) (Melenk and Babuška, 1996) [103];
- "Local Petrov–Galerkin" (MLPG) and "local boundary integral equation" (LBIE) methods (Atluri and Zhu, 1998; Atluri et al., 1999) [104,105];
- "Method of finite spheres" (De and Bathe, 2000) [106];
- "Finite point method" (Oñate et al., 1996; Sulsky and Schreyer, 1996) [107,108];
- "Natural element method" (NEM) (Sukumar et al., 1998) based on a Voronoi tessellation of a set of nodes [109].

Naturally, the main advantage of the meshless approaches is the much reduced demand for meshing compared with standard FEM and FDM/FVM for both continuous and fractured bodies. They can still be viewed as classes of weighted residual techniques in computational mechanics like FEM and BEM, and are performed with three key operations: interpolation using trial (shape) functions; integration to derive governing algebraic equations; and solution of the final system equations. There are basically three interpolation techniques: wavelets, moving least-square functions, and the partition of unity or hp-clouds. The moving least-square techniques are special cases of the partition of unity. The interpolation functions produced using these techniques are non-polynomial functions and this makes the integration of the weak form more demanding compared with standard FEM.

Another shortcoming of many meshless approaches is the difficulty in enforcement of essential boundary conditions. The Kronecker delta function property in the FEM/BEM shape functions ensures that the essential boundary functions are met efficiently. However, in many meshless methods, the generated interpolation functions do not have the Kronecher delta property at nodes, and special techniques must be applied to overcome this difficulty, such as coupling with FEM at boundaries (Krongauz and Belytschko, 1996) [110], Lagrange multipliers (Belytschko et al., 1994) [96], penalty factors (Atluri and Zhu, 1998) [104], etc.

The early meshless formulations, such as diffuse elements, EFG, hp-clouds, PUM, RKPM, and MLSPK are not true meshless techniques because a background mesh (cell) structure is still needed for integration, although not for interpolation. An example of such a cell–node structure in EFG is given in Fig. 12a. The discretization is therefore not adequately 'free'.

The finite point method uses a weighted least-square interpolation (therefore no element structure needed) and point collocation (thus by-passing the elements for integration), and therefore is a true meshless formulation. However, care needs to be taken in the choice of collocation points to ensure correct solutions of the derived equations. Similar formulations are also reported by Zhang et al. (2001) [111].

The MLPG and LBIE methods are true meshless techniques without using background meshes for either interpolation or integration. The trial functions are generated using moving least squares, partition of unity or Shepard functions with the shapes of circles, rectangles or ellipse at the bases. The test functions are the same as the trial functions for the MLPG and the fundamental solutions for the LBIE, respectively, over their respective support and neighbouring nodes of influences (Fig. 12b). The integration is performed over sub-domains defining the supports for the test functions (Ω_{te} in Fig. 12b) or their intersections. A panel factor is used in MLPG to ensure the satisfaction of the essential boundary conditions.

The finite sphere method (De and Bathe, 2000) [106] may be viewed as a special class of MLPG with circular support domain shapes. The difference between the MLPG and the method of finite spheres is the numerical integration scheme. The method of finite spheres uses Gaussian product rules on 2-D annuli and annular sectors with a larger number of integration points compared with the FEM. A similar development by Atluri and Li (2001) [112], called the finite cloud method, uses the combined point collocation and fixed least-square kernel technique for constructing interpolation functions.

The meshless approach greatly simplifies the task of mesh generation in FEM since no fixed elements are required and largely eliminates the element locking effect, with the cost of more computational effort in generating numerically the trial functions over the selected node clusters. From the pure computing performance point of view, it has not yet outperformed the FEM techniques, but it has potential for civil engineering problems in general, and rock mechanics applications in particular, due to its flexibility in treatment of fractures, as reported by Zhang et al. (2000) [113] for analysis of jointed rock masses with block-interface models, and Belytschko et al. (2000) [114] for fracture growth in concrete. Its development was stimulated largely for simulating the mechanics of fractured rocks—because of the latter's unusual complexity in geometry and behaviour of the hosted fractures. A contact-detection algorithm using the meshless technique was also reported by Li et al. (2001) [115] that may pave the way to extending the meshless technique to discrete block system modelling. The concept was also extended to the BEM (see Section 3.3).

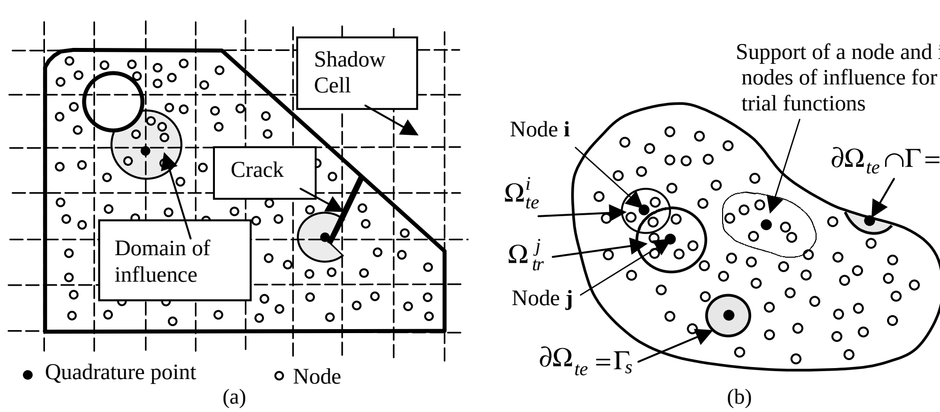

Fig. 12. (a) The cell-and-node structure of the EFG meshless method (Belytschko et al., 1994) [96]; and (b) node and support structure in MLPG and LBIE where symbols Ω^i_{te} and Ω^j_{tr} indicate the support domains of nodes i and j, respectively, and $\partial\Omega_{te}$ indicates the boundaries of the support domains.

3.3. Boundary Element Methods

3.3.1. Basic concepts

Unlike the FEM and FDM methods, the BEM approach initially seeks a weak solution at the global level through an integral statement, based on Betti's reciprocal theorem and Somigliana's identity. For a linear elasticity problem with domain Ω, boundary Γ of unit outward normal vector n_i, and constant body force f_i, for example, the integral statement is written as

$$c_{ij}u_j + \int_{\Gamma} t^*_{ij}u_j \, \mathrm{d}\Gamma = \int_{\Gamma} u^*_{ij}t_j \, \mathrm{d}\Gamma + \int_{\Gamma} \frac{\partial u^*_{ij}}{\partial n} f_j \, \mathrm{d}\Gamma, \tag{8}$$

where u_j and t_j are the displacement and traction vectors on the boundary Γ, the terms u^*_{ij} and t^*_{ij} are called displacement and traction kernels. The term c_{ij} is called the free term determined by the local geometry of the boundary surfaces, $c_{ij}=1$ when the field point is inside the domain Ω.

The solution of the integral Eq. (8) requires the following steps:

(1) *Discretization of the boundary Γ with a finite number of boundary elements*. For 2-D problems, the elements are 1-D line segments which may have one node at the centre of the element (constant element), two nodes at the two ends of the line segment (linear elements) or three nodes with two end nodes and one central node (quadratic elements). Let N denote the total number of boundary elements. The boundary integral equation then is re-arranged into a sum of local integrals over all elements

$$c_{ij}u_j + \sum_{k=1}^{N} \int_{\Gamma_k} t^*_{ij}u_j \, \mathrm{d}\Gamma = \sum_{k=1}^{N} \int_{\Gamma_k} u^*_{ij}t_j \, \mathrm{d}\Gamma + \sum_{k=1}^{N} \int_{\Gamma_k} \frac{\partial u^*_{ij}}{\partial n} f_j \, \mathrm{d}\Gamma. \tag{9}$$

(2) Approximation of the solution of functions locally at boundary elements by (trial) shape functions, in a similar way to that used for FEM. The difference is that only 1-D shape functions with intrinsic coordinate $-1 \leqslant \xi \leqslant 1$ is needed for 2-D BEM problems, and 2-D shape functions with two intrinsic coordinates $-1 \leqslant \xi \leqslant 1$ and $-1 \leqslant \eta \leqslant 1$ are needed for 3-D problems. The displacement and traction functions within each element are then expressed as the sum of their nodal values of the element nodes:

$$u_i = \sum_{k=1}^{m} N_k u_i^k, \quad t_i = \sum_{k=1}^{m} N_k t_i^k, \tag{10}$$

where m is the element order (=1, 2 or 3 for 2-D problems, for example), and u_i^k and t_i^k are the nodal displacement and traction values at node k, respectively.

Substitution of Eqs. (10) into (9) and for

$$T_{ij} = \int_{\Gamma_k} t^*_{ij}N_j \, \mathrm{d}\Gamma, \quad U_{ij} = \int_{\Gamma_k} u^*_{ij}N_j \, \mathrm{d}\Gamma,$$
$$B_i = \int_{\Gamma_k} f_j \frac{\partial u^*_{ij}}{\partial n} \, \mathrm{d}\Gamma \tag{11}$$

Eq. (8) can be written in matrix form as

$$\underset{(2N\times 2N)}{[T_{ij}(l,k)]}\,\underset{(2N\times 1)}{\{u_j(k)\}} = \underset{(2N\times 2N)}{[U_{ij}(l,k)]}\,\underset{(2N\times 1)}{\{t_j(k)\}} + \underset{(2N\times 1)}{\{B_i(k)\}}, \tag{12}$$

where $i,j=1$, 2 for 2-D and 1, 2, 3 for 3-D problems, respectively, $l,k=1,2,\ \ldots,\ N$, and

$$T_{ij}(l,k) = c_{ij}\delta_{lk} + \int_{\Gamma_k} t^*_{ij}N_j \, \mathrm{d}\Gamma. \tag{13}$$

(3) *Evaluation of the integrals T_{ij}, U_{ij} and B_i with point collocation method by setting the source point P at all boundary nodes successively*. Closed-form solutions exist only for some particular cases (see, for examples, Fratantonio and Rencis, 2000; Carini et al., 1999 [116,117]), and numerical integration using Gaussian quadrature is often used. Note that singularity occurs in the above integrals when the source and field points are located on the same elements, and special integration schemes need to be used to evaluate them in a Cauchy Principal Value sense.

(4) *Incorporation of boundary conditions and solution.* Incorporation of the boundary conditions into the matrix Eq. (12) will lead to final matrix equation

$$[A]\{x\} = \{b\}, \tag{14}$$

where the global matrix $[A]$ is a mixture of T_{ij} and U_{ij}, the unknown vector $\{x\}$ is a composite of both unknown displacements and unknown boundary tractions, and the known vector $\{b\}$ is the sum of the body force vector $\{B_i\}$ and the products of T_{ij} with known displacements and U_{ij} with known tractions, respectively. The resultant Eq. (14) is usually fully populated and asymmetric, leading to fewer choices for efficient equation solvers, compared with the sparse and symmetric matrices encountered in the FEM. The solution of Eq. (14) will yield the values of unknown displacements and tractions at boundary nodes. Therefore all boundary values of displacements and tractions are obtained.

(5) *Evaluation of displacements and stresses inside the domain*. For practical problems, it is often the stresses and displacements at some points inside the domain of interest that have special significance. Unlike the FEM in which the desired data are automatically produced at all interior and boundary nodes, whether some of them are needed or not, in BEM the displacement and stress values at any interior point, P, must be evaluated

separately by

$$u_i(P) = -\sum_{k=1}^{M} \hat{T}_{ij}\bar{u}_j^k + \sum_{k=1}^{M} \hat{U}_{ij}\bar{t}_j^k + \sum_{k=1}^{M} \hat{B}_k, \tag{15}$$

$$\sigma_{ij}(P) = -\sum_{l=1}^{M} S_{kij}\bar{u}_k^l + \sum_{l=1}^{M} D_{kij}\bar{t}_k^l, \tag{16}$$

where kernels $\hat{T}_{ij}$, $\hat{U}_{ij}$, S_{kij}, D_{kij} and $\hat{B}_i$ must be re-evaluated according to the new position of the source point inside the domain (closed-form formulae for them are available in many text books on the BEM), usually without singularities unless the point is very close to boundary, and $\bar{u}_j^k$ and $\bar{t}_j^k$ are known or calculated displacement and traction vectors at all boundary nodes.

The boundary integral equation method was used for the first time by Jaswon (1963) and Symm (1963) [118,119] for solving potential problems. The breakthrough for stress analysis in solids came with the work of Rizzo (1967), Cruse and Rizzo (1968) and Cruse (1973), and Cruse (1978) [120–123] for fracture mechanics applications, based on Betti's reciprocal theorem (Betti, 1872) [124] and Somigliana's identity in elasticity theory (Somigliana, 1885) [125]. The basic equations can also be derived using the weighted residual principle, as presented in Brebbia and Dominguez (1977) [126]. The introduction of isoparametric elements using different orders of shape functions in the same fashion as that in FEM, by Lachat and Watson (1976) and Watson (1979) [127,128], greatly enhanced the BEM's applicability for stress analysis problems.

All these works have established the status of BEM as an efficient numerical method for solving general engineering mechanics problems. The most notable original developments of BEM application in the field of rock mechanics may be attributed to Crouch and Fairhurst (1973), Brady and Bray (1978) and Crouch and Starfield (1983) [129–131], quickly followed by many as reported in the rock mechanics and BEM works (Hoek and Brown, 1982; Brebbia, 1987; Pande et al., 1990; Beer and Watson, 1992) [132,133,47,49] for general stress and deformation analysis for underground excavations, soil-structure interactions, groundwater flow and fracturing processes, and a large number of journal and conference publications. Notable examples are the work for stress/deformation analysis of underground excavations with or without faults (Venturini and Brebbia, 1983; Beer and Pousen, 1995a, b; Kayupov and Kuriyagama, 1996; Cerrolaza and Garcia, 1997; Pan et al., 1998; Shou, 2000) [133–139], dynamic problems (Tian, 1990; Siebrits and Crouch, 1993; Birgisson and Crouch, 1998) [140–142], in situ stress and elastic property interpretation (Wang and Ma, 1986; Jing 1987) [143,144], and borehole tests for permeability measurements (Lafhaj and Shahrour, 2000) [145]. Since the early 80s, an important developmental thrust concerns BEM formulations for coupled thermo-mechanical and hydro-mechanical processes, such as the work reported in Pan and Maier (1997), Elzein (2000) and Ghassemi et al. (2001) [146–148]. Due to the BEM's advantage in reducing model dimensions, 3-D applications are also reported, especially using DDM for stress and deformation analysis, such as Kuriyama and Mizuta (1993), Kuriyama et al. (1995) and Cayol and Cornet (1997) [149–151].

The main advantage of the BEM is the reduction of the computational model dimension by one, with much simpler mesh generation and therefore input data preparation, compared with full domain discretization methods such as the FEM and FDM. Using the same level of discretization, the BEM is often more accurate than the FEM and FDM, due to its direct integral formulation. In addition, solutions inside the domain are continuous, unlike the pointwise discontinuous solutions obtained by the FEM and FDM groups. The solution domains of BEM can be divided into several sub-domains with different material properties, and this will often reduce the calculation time as well. The method is also suitable for considering infinite domains (full or half space/plane), due to its use of the fundamental solutions.

However, in general, the BEM is not as efficient as the FEM in dealing with material heterogeneity, because it cannot have as many sub-domains as elements in the FEM. The BEM is also not as efficient as the FEM in simulating non-linear material behaviour, such as plasticity and damage evolution processes, because domain integrals are often presented in these problems. The BEM is more suitable for solving problems of fracturing in homogeneous and linearly elastic bodies.

The BEM formulation described above is called the direct formulation in which the displacements and tractions in the equations have clear physical meanings, are the basic unknowns of the boundary integral equations which are explicitly described on the problem boundary, and can be directly obtained by the solution of the integral equations. In the indirect formulation, on the other hand, the basic unknowns have no physical meanings and are just fictitious source densities related to the physical variables such as displacements and tractions.

The typical indirect BEMs are the Displacement Discontinuity Method (DDM) by Crouch (1976) [152] for 2-D problems and Weaver (1977) [153] for 3-D problems and the Fictitious Stress Method by Crouch and Starfield (1983) [131]. The basic concept of the indirect approach is to place the finite domain of interest into an imaginary infinitely large domain (full or half-plane or spaces) to derive the boundary integral equations relating the physical variables, such as displacements and tractions, to fictitious source densities, such as fictitious load (stress) or displacement discontinuity. Dunbar (1985) [154] showed the equivalence between the direct and indirect BEM approaches.

3.3.2. Fracture analysis with BEM

To apply standard direct BEM for fracture analysis, the fractures must be assumed to have two opposite surfaces, except at the apex of the fracture tip where special singular tip elements must be used. Denote Γ_c as the path of the fractures in the domain Ω with its two opposite surfaces represented by Γ_c^+ and Γ_c^-, respectively, Somigliana's identity (when the field point is on the boundary) can be written as

$$u_j - c_{ij}\Delta u_j + \int_\Gamma t_{ij}^* u_j \,\mathrm{d}\Gamma + \int_{\Gamma_c} t_{ij}^* \Delta u_j \,\mathrm{d}\Gamma = \int_\Gamma u_{ij}^* t_j \,\mathrm{d}\Gamma + \int_{\Gamma_c} u_{ij}^* \Delta t_j \,\mathrm{d}\Gamma + \int_\Gamma \frac{\partial u_{ij}^*}{\partial n} f_j \,\mathrm{d}\Gamma, \tag{17}$$

where Δu_i and Δt_i are the displacement and traction jumps across the two opposite surfaces of the fractures.

Because of the very small thickness of fractures, the two nodes at the opposite surfaces of a fracture will in fact occupy the same coordinates. This will naturally lead to singular global stiffness matrices if the same boundary conditions (or unknowns) are specified at the two opposite fracture surfaces. Also, any set of equal and opposite tractions on the fracture surfaces will lead to the same equation since $\Delta t_i = 0$. In addition, the displacement difference Δu_j becomes an additional unknown on Γ_c besides u_j.

To overcome these difficulties, two techniques were proposed. One was to divide the problem domain into multiple sub-domains with fractures along their interfaces (Fig. 13a), by Blandford et al. (1981) [155]. This way, the stiffness matrices contributed by opposite surfaces of the same fracture will belong to different sub-domain stiffness matrices; thus, the singularity of the global matrix is avoided. This technique, however, requires the knowledge of fracturing paths (used for deciding sub-regions) and growth rate (for deciding element sizes), which is determined by the solution of the problem itself, before the problem solution, and may not be applicable for many practical problems without symmetry in geometry and boundary conditions.

The second technique is the Dual Boundary Element Method (DBEM). The essence of this technique is to apply displacement boundary equations at one surface of a fracture element and traction boundary equations at its opposite surface, although the two opposing surfaces occupy practically the same space in the model. The general mixed mode fracture analysis can be performed naturally in a single domain (Fig. 13b). The term DBEM was first presented in Portela (1992) and Portela et al. (1992, 1993) [156–158], and was extended to 3-D crack growth problems by Mi and Aliabadi (1992, 1994) [159,160]. However, the original concept of using two independent boundary integral equations for fracture analysis, one displacement equation and another its normal derivative, was developed first by Watson (1979) [128]. Special crack tip elements, such as developed in Yamada et al. (1979) and Aliabadi and Rooke (1991) [161,162], are used at the fracture tips to account for the stress and displacement singularity.

The DDM has been widely applied to simulate fracturing processes in fracture mechanics in general and in rock fracture propagation problems in particular due to the advantage that the fractures can be represented by single fracture elements without need for separate representation of their two opposite surfaces, as should be done in the direct BEM solutions. It was developed by Crouch and Starfield (1983) [131] with open fractures, and was extended to fractures with contact and friction by Wen and Wang (1991) [163] and Shen (1991) [164] for mechanical and rock engineering analyses, respectively. The fictitious unknowns are the displacement discontinuity Δu_i acting on the boundary Γ of a finite body of domain Ω, inserted in an infinitely large half (or full) space. The displacement and traction is given by

$$u_i = c_{ij}\Delta u_j + \int_\Gamma u_{ij}^* \Delta u_j \,\mathrm{d}\Gamma + \int_{\Gamma_c} \bar{u}_{ij}^* \Delta u_j \,\mathrm{d}\Gamma,$$
$$t_i = c_{ij}\Delta u_j + \int_\Gamma t_{ij}^* \Delta u_j \,\mathrm{d}\Gamma + \int_{\Gamma_c} \bar{t}_{ij}^* \Delta u_j \,\mathrm{d}\Gamma \tag{18}$$

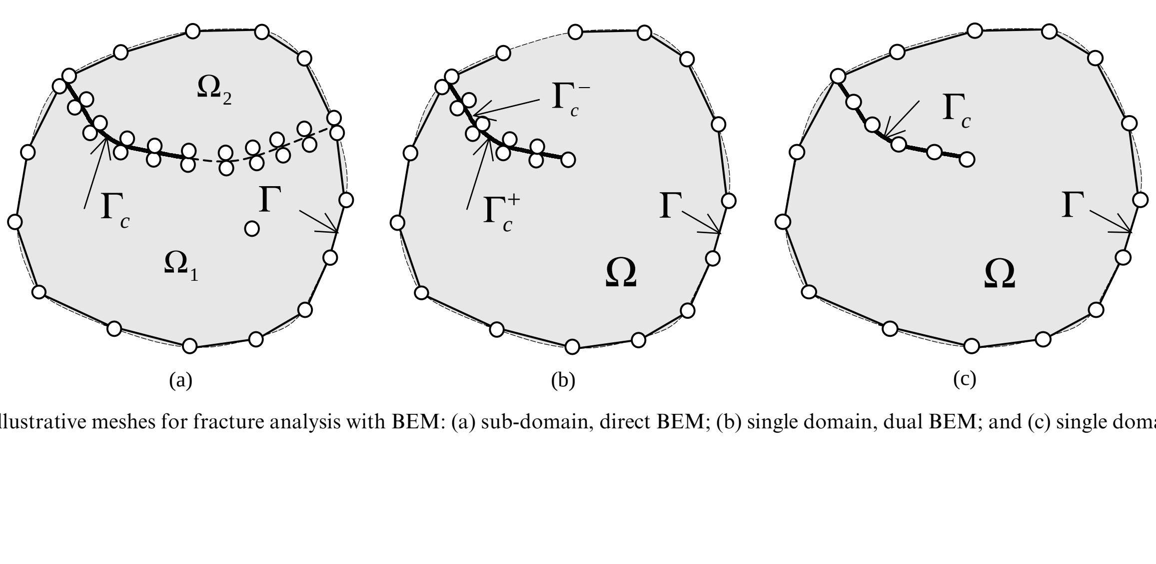

Fig. 13. Illustrative meshes for fracture analysis with BEM: (a) sub-domain, direct BEM; (b) single domain, dual BEM; and (c) single domain DDM.

for field points on Γ. The kernels $\bar{u}^*_{ij}$ and $\bar{t}^*_{ij}$ are the fundamental solutions due to unit discontinuity per unit length (for 2-D) or area (for 3-D), see Appendix A in Wen (1996) [165] for details.

By writing displacement and traction equations in Eq. (18) at boundary and fracture nodes with corresponding known displacements and tractions, respectively, a determinate matrix equation can be produced and its solution will yield all discontinuities Δu_i at all boundary and fracture nodes, which can then be used to produce displacements or stresses at required points inside domain Ω.

For fracturing analysis using numerical methods, there are two main tasks, besides the establishment of the system equations. They are the evaluation of the stress intensity factors (SIF) and simulating fracture growth, based on determination of fracture deformation modes (I, II and III) and different criteria for fracture growth, such as maximum tensile strength or energy release rate. The details can be obtained in Aliabadi and Rooke (1991) [162], Wen (1996) [165] and Mi (1996) [166], for both displacement discontinuity and DBEM formulations applied for fracture growth analysis. A comprehensive coverage of the subject is given in Alliabadi (1999) [167], together with topics of rock fragmentation using DFEM and transport using DFN.

Analysing fracturing processes using BEM is challenging, especially for rock mechanics problems. On the one hand, what happens exactly at the fracture tips in rocks still remains to be adequately understood, with the additional complexities caused by the microscopic heterogeneity and non-linearity at the fracture tip scale, especially regarding the fracture growth rate. On the other hand, complex numerical manipulations are still needed for re-meshing following the fracture growth process so that the tip elements are added to where new fracture tips are predicted, and updating of system equations following the re-meshing, although the task is much less cumbersome than that required for domain discretization methods such as standard FEM.

Due to the above difficulties, fracture growth analyses in rock mechanics have not been widely applied, and were mostly performed with 2-D BEM using indirect formulations such as DDM, considering often a small number of isolated, non-intersecting fractures in usually, small 2-D models concerning local failure mechanisms, such as borehole breakout and mechanical breakage (Tan et al., 1998) [168]. A 3-D DDM code POLY3D is developed at Stanford University, which is able to consider a number of non-intersecting fractures in 3-D (La Pointe et al., 1999) [169]. Usually, fracture growth is ignored completely in rock mechanics applications, and only stress and deformations of large-scale fractures are included, such as faults or fracture zones, using the multi-region formulations (Crotty and Wardle, 1985) [170] and in Beer and Pousen (1995a, b) [134,135]. This can be justified by the fact that the scale of fracture growth, under the normal loading conditions encountered in most civil engineering projects, is small and omitting this factor will not cause major misinterpretations of the behaviour of rock masses under consideration. The assumption, however, will not be true for other problems—such as borehole stability and rock spalling, where the former is dominated by in situ stresses, and the latter is controlled by the local stresses, local fracture geometry and dynamic fracturing processes.

The effects of fluid pressures and heat gradients on fracturing process in rock, either static or dynamic, is not properly understood, even less analysed by numerical methods, due to the added complexity in the physics, equations and numerical solution procedures. However, considering their potential significance for the performance and safety of many environment-oriented civil engineering projects in fractured rocks, the need for such studies is clear.

3.3.3. Alternative formulations associated with BEM

The standard BEM, dual BEM and DDM as presented above have a common feature: the final coefficient matrices of the equations are fully populated and asymmetric, due to the traditional nodal collocation technique. This makes the storage of the global coefficient matrix and solution of the final equation system less efficient, compared with FEM. They suffer also from another drawback: the need for special treatment of sharp corners on the boundary surfaces (curves) or at the fracture intersections, because of the change of directions of the outward unit normal vectors at the corners. This causes an inadequate number of equations in the final system compared with that of the degrees of freedom. Artificial corner smoothing, additional nodes or special corner elements are usually the techniques applied to solve this particular difficulty. In a special formulation of BEM, called the Galerkin BEM, or Galerkin Boundary Element Method (GBEM), these two shortcomings are removed automatically as the consequences of the Galerkin formulation.

3.3.3.1. Galerkin Boundary Element Method. The GBEM produces a symmetric coefficient matrix by multiplying the traditional boundary integral by a weighted trail function and integrates it with respect to the source point on the boundary for a second time, in a Galerkin sense of weighted residual formulation. For an elasticity problem, the Somigliana identity becomes a double integral equation

$$\int_\Gamma w_i c_{ij} u_j \, d\Gamma + \int_\Gamma \int_\Gamma w_i t^*_{ij} u_j \, d\Gamma \, d\Gamma = \int_\Gamma \int_\Gamma w_i u^*_{ij} t_j \, d\Gamma \, d\Gamma + \int_\Gamma w_i \frac{\partial u^*_{ij}}{\partial n} f_j \, d\Gamma, \tag{19}$$

where w_i is the weight function. Discretizing the boundary into elements and choosing the weight functions the same as the trial (shape) functions $\{N_u\}$ for displacement and $\{N_t\}$ for tractions, respectively, in vector form and using the Galerkin approximation technique, the boundary integral equation in Eq. (19) becomes a matrix equation of the form

$$\begin{bmatrix} [C^{uu}] & -[C^{ut}] & [C^{uc}] \\ -[C^{tu}] & [C^{tt}] & -[C^{tc}] \\ [C^{cu}] & -[C^{ct}] & [C^{cc}] \end{bmatrix} \begin{Bmatrix} \{t\} \\ \{u\} \\ \{\Delta u\} \end{Bmatrix} = \begin{Bmatrix} \{f^u\} \\ -\{f^t\} \\ \{f^c\} \end{Bmatrix}, \tag{20}$$

where the vectors $\{t\}, \{u\}$and $\{\Delta u\}$ are the unknown traction and displacement vectors at the boundary and displacement discontinuity vector along the fractures inside the domain, respectively. The right-hand vectors $\{f^u\}, \{f^t\}$and $\{f^c\}$ are obtained from known boundary displacement and traction conditions on the boundary and loading condition along the fractures, respectively. The coefficient sub-matrices $[C^{mn}]$ ($m, n = u, t$, and c) are contributions from double integrals over elements with specified displacements (Γ_u), traction (Γ_t), or along the fracture paths (Γ_c), respectively, written as (Carini et al., 1999) [117]

$$[C^{uu}] = \int_{\Gamma_u}\int_{\Gamma_u} \{N_t(Q)\}^{\mathrm{T}}[G^{uu}(P,Q)]\{N_t(Q)\}\,\mathrm{d}\Gamma_{(Q)}\,\mathrm{d}\Gamma_{(P)}, \tag{21a}$$

$$[C^{ui}] = \int_{\Gamma_i}\int_{\Gamma_u} \{N_t(Q)\}^{\mathrm{T}}[G^{ut}(P,Q)]\{N_t(Q)\}\,\mathrm{d}\Gamma_{(Q)}\,\mathrm{d}\Gamma_{(P)}, \quad (i = t, c) \tag{21b}$$

$$[C^{ij}] = \int_{\Gamma_i}\int_{\Gamma_j} \{N_u(Q)\}^{\mathrm{T}}[G^{tt}(P,Q)]\{N_u(Q)\}\,\mathrm{d}\Gamma_{(Q)}\,\mathrm{d}\Gamma_{(P)} \quad (i,j = t, c), \tag{21c}$$

$$\begin{aligned} \{f^u\} &= \int_{\Gamma_u} \{N_t\}\{\bar{f}\}^{\mathrm{T}}\,\mathrm{d}\Gamma_u, \\ \{f^t\} &= \int_{\Gamma_t} \{N_u\}\{\bar{f}_t\}^{\mathrm{T}}\,\mathrm{d}\Gamma_t, \\ \{f^c\} &= \int_{\Gamma_c} \{N_u\}\{\bar{f}_c\}^{\mathrm{T}}\,\mathrm{d}\Gamma_c, \end{aligned} \tag{21d}$$

where elements in the matrices $[G^{mn}]$ ($m, n = u, t$, and c) are kernel functions derived from fundamental solutions of the problems. Due to the extra integration, the singularity of these kernels increases, and the kernels $[C^{ut}]$ and $[C^{tu}]$ are called strongly singular kernels whose integration must be evaluated in a Cauchy principal value sense and kernel $[C^{pp}]$ is called hypersingular kernels which must be evaluated in the Hadamard finite parts.

The GBEM is an attractive approach due to the symmetry of its final system equation, which paves the way for the variational formulation of BEM for solving non-linear problems. It is also flexible in choosing different formulations, and the traditional BEM can be seen as a special case. A recent review on GBEM is given by Bonnet et al. (1998) [171] in which the theoretical foundation and applications in different mechanics fields are summarized. However, integration of these strong and hypersingular kernels are more difficult than the common singular kernels in the traditional BEM. Analytical integration techniques were also proposed for elements with low order shape functions (see Salvadori, 2001) [172], and singularity subtraction techniques were also proposed to ease the task of integrations (Michael and Barbone, 1998) [173]. Applications of GBEM into rock mechanics problems has commenced (Wang et al., 2001) [174].

3.3.3.2. Boundary Contour Method. The Boundary Contour Method (BCM) involves rearranging the standard BEM integral Eq. (8) so that the difference of the two integrals appearing on the right-hand side of Eq. (8) can be represented by a vector function $F_i = u^*_{ij}t_j - t^*_{ij}u_j$ which is divergence free, i.e. $\nabla \cdot \mathbf{F} = 0$, except at the point of singularity (Nagarajan et al., 1994, 1996). This property of F_i ensures the existence of a vector function V_i so that $\mathbf{F} = \nabla \times \mathbf{V}$ and the BEM equation can be rewritten as

$$c_{ij}u_j = \int_{\Gamma} (u^*_{ij}t_j - t^*_{ij}u_j)\,\mathrm{d}\Gamma = \int_{\Gamma} F_i\,\mathrm{d}\Gamma = \oint_{\partial\Gamma} V_i\,\mathrm{d}l, \tag{22}$$

where $\partial\Gamma$ is the boundary of the boundary Γ of the domain Ω of interest. Therefore, if the vector potential function V_i can be obtained, the dimension of the computational model can be reduced further by one, i.e. it is only necessary to evaluate values of V_i at nodes for 2-D BEM without integration, and integrals along the closed contours of psuedo-2-D elements for 3-D BEM, thus explaining the name BCM.

Since the vector function F_i contains the unknown fields of displacements and tractions, special shape functions must be chosen for them to determine the potential function V_i. For 2-D linear elasticity problems, a five node quadratic element with two traction nodes and three displacement nodes with equal intervals in between (Fig. 14a) is used to completely define the shape functions. Three different kinds of 3-D elements were proposed for 3-D elasticity problems, and the simplest one is a four-node triangle element with three displacement nodes and one traction node at the centre (Fig. 14b). The values of vector function V_i will then be determined based on the shape functions (in closed-form for 2-D problems and using numerical integrations for 3-D problems), which are then used to determine

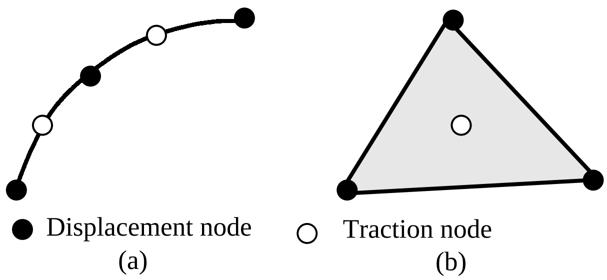

Fig. 14. (a) Boundary elements used for 2-D BCM and (b) 3-D BCM (after Nagarajan et al., 1996) [176].

nodal values of boundary displacements and tractions. Details for the numerical implementation of BCM can be seen in Nagarajan et al. (1994, 1996), Phan et al. (1997), Mukherjee et al. (1997) and Zhou et al. (1999) [175–179]. The method has also been combined with Galerkin approximation, to become the so-called Galerkin Boundary Contour Method (GBCM) (Zhou et al., 1998; Novati and Springhetti, 1999, [180,181]), and generated a symmetric final coefficient matrix when all elements are straight segments for 2-D problems. Similar developments, combined with meshless approaches, are also reported in Chati et al. (2001) [182].

The BCM approach is attractive mainly because of its further reduction of computational model dimensions and simplification of the integration. The savings in pre-processing of the simulations are clear. Treatment of fractures and material non-homogeneity has not been studied in BCM; these may limit its applications to rock mechanics problems considering the present state-of-the-art.

3.3.3.3. Boundary Node Method. Another recent development associated with the BEM is the Boundary Node Method (BNM) (Mukherjee and Mukherjee, 1997; Chati et al., 1999; Chati and Mukherjee, 2000; Kothnur et al., 1999) [183–186]. The method is a combination of traditional BEM with a meshless technique using the moving least squares for establishing trial functions without an explicit mesh of boundary elements. It further simplifies the mesh generation tasks of the BEM at the cost of increasing computational operations for establishing the trial functions. Its applications concentrate on shape sensitivity analysis at present and solution of potential problems (Gu and Liu, 2002; Gowrishankar and Mukherjee, 2002) [187,188], but can be extended to general geomechanics problems, especially groundwater flow and stress/deformation analysis.

3.3.3.4. Dual Reciprocity Boundary Element Method (DRBEM). When source terms are presented in BEM formulations, such as gravitational body forces, heat sources, sinks/source terms in flow problems, thermal stress fields, etc., domain integrals often appear in the formulation. This problem will also occur when considering initial stress/strain effects, and non-linear material behaviour such as plastic deformation. The traditional technique to deal with such domain integrals is the division of the domain into a number of internal cells, which will seriously compromise the advantages of BEM's "boundary only" discretization. Different techniques have been developed over the years to overcome this difficulty (see for example Brebbia et al., 1984; Partridge et al., 1992) [189,190], as listed below.

- Analytical integration of domain integrals, which is applicable to limited cases of simple geometry and boundary conditions.
- Fourier expansion of integrand functions, which is limited by domain geometry and boundary conditions, with additional computational cost of calculating the expansion coefficients.
- Galerkin vector technique based on Green's identity and higher order fundamental solutions, which can be readily applied to transform the domain integrals into boundary ones when the source terms are simple, such as constant body forces or heat sources. When the source terms are complex functions of space and time, the higher fundamental solutions may be difficult to obtain and the technique cannot be effectively used. An extension of this technique using multiple higher order fundamental solutions, instead of just one as in the Galerkin vector method, is called the Multiple Reciprocity Method in the literature.
- The Dual Reciprocity Method (DRM), which is a more generalized method closely related to the Galerkin vector and multiple reciprocity techniques (MDR) for constructing particular solutions suitable for non-linear and time-dependent problems. The source terms can be more generalized functions of space and time. Global interpolation functions, or Radial basis functions, are often used in converting the domain integrals into boundary ones (Cheng et al., 1994) [191].

The numerical treatment of the domain integrals, when the initial domain fields must be considered, is a significant subject in BEM, since it relates to some important rock mechanics problems as mentioned above. The DRM approach appears to be the most widely applied technique so far since it has a unified approach to treat different source terms and initial fields. A revisit of the technique for thermo-elasticity and elastic body force problems is presented in Cheng et al. (2001) [192] and an application for groundwater flow problem is presented in El Harrouni et al. (1997) [193]. Treatment of initial stress and strain fields related to plastic deformation is presented in Ochiai and Kobayashi (1999, 2001) and Gao (2002) for both 2-D and 3-D elasto-plastic problems [194–196]. Despite the efforts, the numerical efficiency of the DRM is still a subject of debate and has significant influence on the suitability and efficiency of BEM for problems of material non-homogeneity and non-linearity.

The BEM methods as a whole is less efficient in treatment of material non-homogeneity and non-linearity compared with FEM and FVM, despite the fact that a limited number of multiple sub-regions of different material properties can be efficiently handled in BEM. However, it is much more efficient in simulating fracturing process (initiation, growth and coalescence) in elastic solids. It is therefore not surprising that BEM is most often applied for stress analysis problems, or fracturing process simulation, of CHILE continua, and is used for far-field representations in hybrid models.

3.4. Basic features of the Discrete Element Method (DEM)

Rock mechanics is one of the disciplines from which the DEM originated (Burman, 1971; Cundall, 1971, 1974; Chappel, 1972, 1974; Byrne, 1974) [197–202]. The other engineering branches that stimulated the development of the DEM are structural analysis and multi-body systems. The theoretical foundation of the method is the formulation and solution of equations of motion of rigid and/or deformable bodies using implicit (based on FEM discretization) and explicit (using FVM discretization) formulations. The method has a broad variety of applications in rock mechanics, soil mechanics, structural analysis, granular materials, material processing, fluid mechanics, multi-body systems, robot simulation, computer animation, etc. It is one of most rapidly developing areas of computational mechanics. The key concept of DEM is that the domain of interest is treated as an assemblage of rigid or deformable blocks/particles/bodies and the contacts among them need to be identified and continuously updated during the entire deformation/motion process, and represented by proper constitutive models. This fundamental conception leads naturally to three central issues:

(i) identification of block or particle system topology based on the fracture system geometry, or particle shape assumptions within the domain of interest;
(ii) formulation and solution of equations of motion of the block (particle) system;
(iii) detection and updating of varying contacts between the blocks (particles) as the consequences of motions and deformations of the discrete system.

The basic difference between DEM and continuum-based methods is that the contact patterns between components of the system are continuously changing with the deformation process for the former, but are fixed for the latter.

To formulate a DEM method to simulate the mechanical processes in rock mechanics applications, the following problems must be solved:

(1) space sub-division and identification of block system topology;
(2) representation of block deformation (rigid or deformable, using FVM or FEM);
(3) developing an algorithm for contact detection (penalty function, Lagrange multiplier, or augmented Lagrange multiplier);
(4) obtaining constitutive equations for the rock blocks and fractures;
(5) integration of the equations of motion of the blocks/particles (dynamic relaxation; time-marching FVM).

The block system identification depends on the reconstruction of the fracture system in situ according to usually very limited data from borehole logging or surface/underground mapping of fracture systems at generally very limited exposure areas. The usual procedure is to establish probabilistic density functions (PDFs) of the fracture parameters (orientation, frequency, size/trace length, etc.) and then use the random number field technique to re-generate a number of realizations of synthetic fracture systems that share the same statistical geometric properties of the sampled fracture population from the logging and mapping. The reliability of such stochastic models of fracture system is therefore dependent on the quality of the logging and mapping operations that, in turn, depends on the quantity of data (therefore areas for fracture mapping and numbers/length of boreholes) available. It is evident that reliability of such fracture system models are largely unknown or the level of uncertainty is very high, due simply to the fact that the real fracture systems are hidden inside rock masses and will never be fully accessed by measurements. Regarding this difficulty, a large number of such realizations need to be generated so that they, collectively, provide a much improved representation of the stochastic nature of the fracture system. This is called Monte Carlo simulation, and will naturally cause significant increase of DEM computational cost.

When the fracture systems are available, either re-generated or assumed, the next step is to construct the block systems defined by the fracture system. The task is trivial if the fractures are infinitely long (or large) and follow regular patterns of distributions (such as constant spacing and fixed orientations). However, for random fracture systems of finite fracture size, special techniques of combinatorial topology is needed to construct the block systems according to the fracture system geometry (see Lin et al., 1987; Lin and Fairhurst, 1988; Lin, 1992; Jing and Stephansson, 1994a, b; Jing, 2000; Lu, 2002) [203–209].

For rigid block analysis, an explicit time-marching scheme is used to solve the dynamic equations of motion of the rigid block system, based on a dynamic or static relaxation scheme, or an FDM approach in the time domain. For deformable block systems, the solution strategies are different for the treatment of block deformability. One is explicit solution with finite volume discretization of the block interiors, without the need for solving large-scale matrix equations. The other is an implicit solution with finite element discretization of the block interiors, which leads to a matrix equation representing the deformability of the block systems, similar to that of the FEM.

The most representative explicit DEM methods is the Distinct Element Method created by Cundall (1980, 1988) [210,211] with the computer codes UDEC and 3DEC for 2- and 3-D problems of rock mechanics (ITSACA, 1992, 1994) [212,213]. Other developments were made in parallel with the distinct element approach and used the name 'discrete element methods', such as in Taylor (1983), Williams et al. (1985), Williams and Mustoe (1987), Williams (1988), Willaims and Pentland (1992), Mustoe (1992), Hocking (1977, 1992), Williams and O'Connor (1995) [214–222]. Another approach, so-called 'Block-Spring Model' (BSM) is essentially a version of DEM with rigid blocks linked by springs and applied for structural (Kawai, 1977a, b; Kawai et al., 1978) [223–225] and rock engineering problems (Wang and Garga, 1993; Wang et al., 1997; Li and Vance, 1999; Hu, 1997; and Li and Wang, 1998) [226–230]. However, the approach and codes by the Distinct Element Methods appears to be the main direction of application in rock mechanics problems, even the term 'discrete element methods' is more universally adopted.

The implicit DEM was represented mainly by the Discontinuous Deformation Analysis (DDA) approach, originated by Shi (1988) [231] and further developed by Shyu (1993) and Chang (1994) [232,233] for stress/deformation analysis, and Kim et al. (2000) and Jing et al. (2001) for coupled stress-flow problems [234,235]. The method uses standard FEM meshes over blocks and the contacts are treated using the penalty method. Similar approaches were also developed by Ghaboussi (1988), Barbosa and Ghaboussi (1989, 1990) [236–238]. The technique uses four-noded blocks as the standard element and is called the Discrete Finite Element Method. Another similar development, called the combined finite-DEM (Munjiza et al., 1995, 1999; Munjiza and Andrews, 2000) [239–241], considers not only the block deformation but also fracturing and fragmentation of the rocks. However, in terms of development and application, the DDA approach occupies the front position. DDA has two advantages over the explicit DEM: permission for relatively larger time steps and closed-form integrations for the stiffness matrices of elements. An existing FEM code can also be readily transformed into a DDA code while keeping all the advantageous features of the FEM.

3.4.1. Explicit DEM—Distinct Element Method: block systems

The Distinct Element Method was originated in the early 70s by a landmark paper on the progressive movements of rock masses as 2-D rigid block assemblages (Cundall, 1971) [198]. The work was extended later into a code, RBM, written in machine language for a NOVA mini-computer (Cundall, 1974) [199]. The method and the RBM code later progressed, firstly by approximating the deformation of blocks of complex 2-D geometry by a constant strain tensor, with the code translated into the FORTRAN language and called SDEM for block systems (Cundall and Marti, 1979) [242]. A separate version of the SDEM code, called CRACK, was created to consider fracturing, cracking and splitting of intact blocks under loading, based on a tensile failure criterion. The representation of "simply deformable blocks" causes incompatibility between the complex block geometry and constant strain tensor, and the difficulty was overcome later by using full internal discretization of blocks by finite volume meshes of triangle elements, leading to early versions of the code UDEC (Cundall, 1980; Cundall and Hart, 1985) [210,243], which has a BEM function representing the far-field (Lemos, 1987) [244]. Extension to 3-D problems was developed by Cundall (1988) [211] and Hart et al. (1988) [245], leading to the code 3DEC.

The technique of the explicit DEM is presented comprehensively in Cundall and Hart (1992), Hart (1993) and Curran and Ofoegbu (1993) [246–248]. The principle of simulating large-scale deformations of elasto-plastic materials using finite difference/volume schemes developed in Wilkins (1963) [16] and the dynamic relaxation principles (Southwell, 1935, 1940, 1956) [249–251] are the mathematical basis. The contact detection and updating is performed based on the "contact overlap" concept. The method and codes were then developed further by coupling heat conduction and viscous fluid flow through fractures (treated as interfaces between block boundaries).

3.4.1.1. Block discretization. Blocks are represented as convex polyhedra in 3-D with each face a planar convex polygon having a finite number of rectilinear edges. Their 2-D counterparts are general polygons with a finite number of straight edges (Fig. 15). The 2-D polygons can be either convex or concave, but the 3-D polyhedral must be convex. These blocks are formed by fractures which are represented in the problem domain either individually (for larger-scale fractures) or by a fracture sets generator (for smaller-scale fracture sets) using random distributions—based on site or modelling requirement data—of dip angles, dip directions, spacing

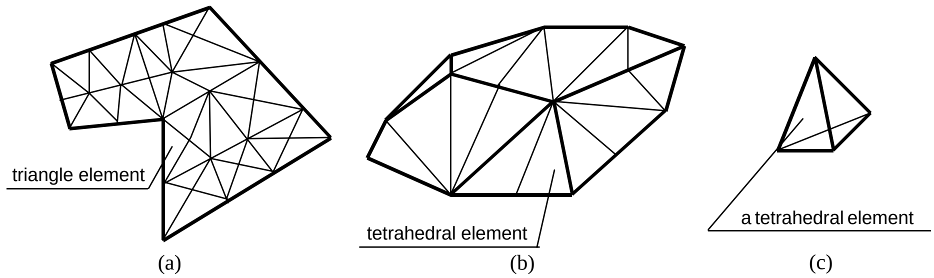

Fig. 15. Discretization of blocks by: (a) constant strain triangles; (b) constant strain tetrahedral; and (c) a typical tetrahedral element.

Table 1
Types of contacts for polygons and polyhedral

Block shapes	Contact types
Arbitrary polygons (convex or concave) (2-D block)	vertex-to-vertex, vertex-to-edge, edge-to-edge
Convex polyhedral (3-D block)	vertex-to-vertex, vertex-to-edge, vertex-to-face, edge-to-edge, edge-to-face, face-to-face

and apertures of the sets. The vertices (corners), edges and faces of individual blocks and their connection relations are identified during the block generation process.

The deformable blocks are further divided into a finite number of constant strain triangles in 2-D or tetrahedra in 3-D. These triangles or tetrahedra form a mesh of the FVM (zones). Rectangular element meshes can also be used for 2-D problems when the problem geometry is favourable.

3.4.1.2. Representation of deformation. An explicit, large strain Lagrangian formulation for the constant strain elements is used to represent the element deformations. The displacement field of each element varies linearly and the faces or edges of the elements remain as planar surface or straight line segments. Higher order elements may also be used, but curved boundary surfaces (or edges) may be obtained, which may in turn complicate the contact-detection algorithm.

Based on Gauss' theorem to convert volume (area) integrals into surface (line) integrals, the increments of element strain can be written

$$\Delta\varepsilon_{ij} \approx \frac{\Delta t}{2}\sum_{k=1}^{N}[(v_i^m)n_j \pm (v_j^m)n_i]\Delta S^k, \tag{23}$$

where ΔS^k is the area (or length) of the kth boundary face (or edge) with unit normal n_i^k, and v_i^m is the mean value of velocity over ΔS^k. The summation extends over the N faces (or edges) of an element (zone). The sign "+" is used if $i = j$; otherwise, the sign "−" is used. Δt is the time step. The stress increments are obtained by invoking the constitutive equations for the block materials.

3.4.1.3. Representation of contacts. Kinematically, block contacts are determined by the smallest distance between two blocks, pre-set in the codes or models. When this distance is within a prescribed threshold, a potential contact between these two blocks is numerically established. The contact-detection algorithm in the Distinct Element Method programs determines the contact type (different touching patterns between vertices, edges and faces), the maximum gap (if two blocks do not touch but are separated by a gap close to the pre-set tolerance), and the unit normal vector defining the tangential plane on which sliding can take place. Table 1 lists all types of contacts.

Mechanically, the interaction between two contacting blocks is characterized by a stiffness (spring) in the normal direction and a stiffness and friction angle (spring-slip surface series) in the tangential directions with respect to the fracture surface (contact plane, see Fig. 16a). Interaction forces developed at contact points are determined as linear or non-linear functions of the deformations of springs and slip surfaces (i.e., the relative movements of blocks at contact points) and resolved into normal and tangential components, depending the constitutive models of the contacts (point contacts or edge/face contacts).

The concept of contact 'overlap', though physically inadmissible in block kinematics—because blocks should not interpenetrate each other—may be accepted as a mathematical means to represent the deformability of the contacts. However, it does present a numerical shortcoming that is difficult to overcome when the normal forces or stresses at contact points are large. In this case, even with high normal stiffness, the 'overlap' may be too excessive to be acceptable and the calculation has to be stopped to implement some remedial measure (for example, to increase the normal

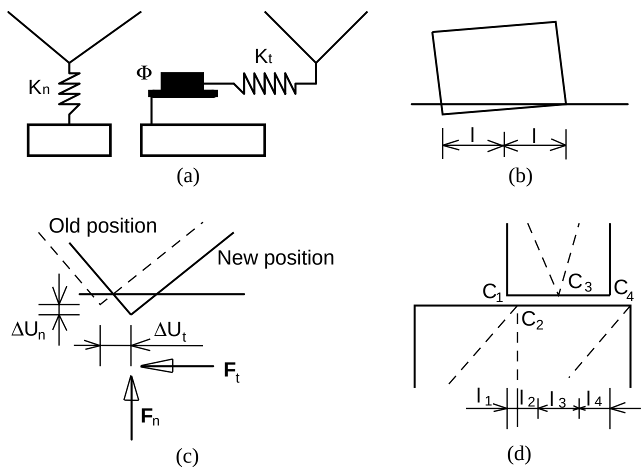

Fig. 16. Mechanical representation of contacts in the 2-D DEM.

stiffness) and start again. It also presents a problem for fluid flow calculation in which the apertures of fractures may become negative if 'overlap' occurs at contact points. The mathematical representation of the contact 'overlap' is thus not fully compatible with physical reality.

3.4.1.4. Numerical integration of the equations of motion. An explicit central difference scheme is applied in the Distinct Element Method to integrate the equations of motion of the block system, as opposed to the implicit approach utilized in other continuum-based numerical methods. The unknown variables (contact forces or stresses) on the block boundary or in the internal elements are determined locally at each time step from the known variables on the boundaries, in the elements and their immediate neighbours. There is no need to set up and solve a matrix form of the equations of motion. The non-linearity in the material behaviour (of the fractures or intact blocks) can be handled in a straightforward manner.

The equations of motion for a rigid block, in terms of translational and rotational velocities, are written

$$\begin{aligned} v_i^{(t+\Delta t/2)} &= v_i^{(t-\Delta t/2)} + \left[\frac{\sum f_i}{m} + b_i\right]\Delta t, \\ \omega_i^{(t+\Delta t/2)} &= \omega_i^{(t-\Delta t/2)} + \frac{\sum M_i}{I}\Delta t, \end{aligned} \tag{24}$$

where m is the block mass, I is the moment of inertia, b_i are the volume force components of the block and M_i are the components of the resultant moment. The displacement at the next time step is then given by

$$\begin{aligned} u_i^{(t+\Delta t)} &= u_i^{(t)} + v_i^{(t+\Delta t/2)}\Delta t, \\ \theta_i^{(t+\Delta t)} &= \theta_i^{(t)} + \omega_i^{(t+\Delta t/2)}\Delta t, \end{aligned} \tag{25}$$

where θ_i is the angular displacement of the block.

For deformable blocks, the equations of motion are written for grid points—the vertices of internal difference elements. The central difference scheme is similar to the first equation in Eq. (24) with only one modification to the resultant out-of-balance force, f_i

$$f_i = f_i^c + \sum_{k=1}^{N} \sigma_{ij}(n_j^k \Delta S^k), \tag{26}$$

where f_i^c is the resultant contact force if the grid point is on the boundary of the block. The symbol N denotes the number of difference elements connected by this grid point.

At each time step, the kinematic quantities (velocities, displacements and accelerations) are first calculated and the contact forces or stresses, as well as the internal stresses of the elements, are then obtained via constitutive relations for contacts.

In the general calculation procedure, two basic tasks are performed in turn. The kinematic quantities are updated first, followed by invoking the constitutive relations to provide the corresponding forces and stresses, see Fig. 17.

3.4.1.5. Applications and remarks. Due mainly to its conceptual attractions in the explicit representation of fractures, the DEM, especially the Distinct Element Method, has been enjoying wide application in rock engineering. A large quantity of associated publications has been published, especially in conference proceedings: it is not practical to list these even at a moderate level for this review. Therefore, a few representative references, mainly in international journals, are given here to show the wide range of the applicability of the methods:

- Tunnelling, underground excavations and mining: Barton (1991), Jing and Stephansson (1991), Nordlund et al. (1995), Chryssanthakis et al. (1997), Hanssen et al. (1993), Kochen and Andrade (1997), McNearny and Abel (1993), Souley et al. (1997a, b), Sofianos and Kapenis (1998) and Lorig et al. (1995) [252–262];
- Rock dynamics: Zhao et al. (1999) and Cai and Zhao (2000) [263,264];
- Nuclear waste repository design and performance assessment: Chan et al. (1995), Hansson et al. (1995), Jing et al. (1995, 1997) [265–268];
- Reservoir simulations: Gutierrez and Makurat (1997) [269];
- Fluid injection: Harper and Last (1989, 1990a, b) [270–272];
- Rock slopes, caving and gravity flow of particle systems: Zhu et al. (1999) [273];
- Laboratory test simulations and constitutive model development for hard rocks: Jing et al. (1993, 1994), Lanaro et al. (1997) [274–276];

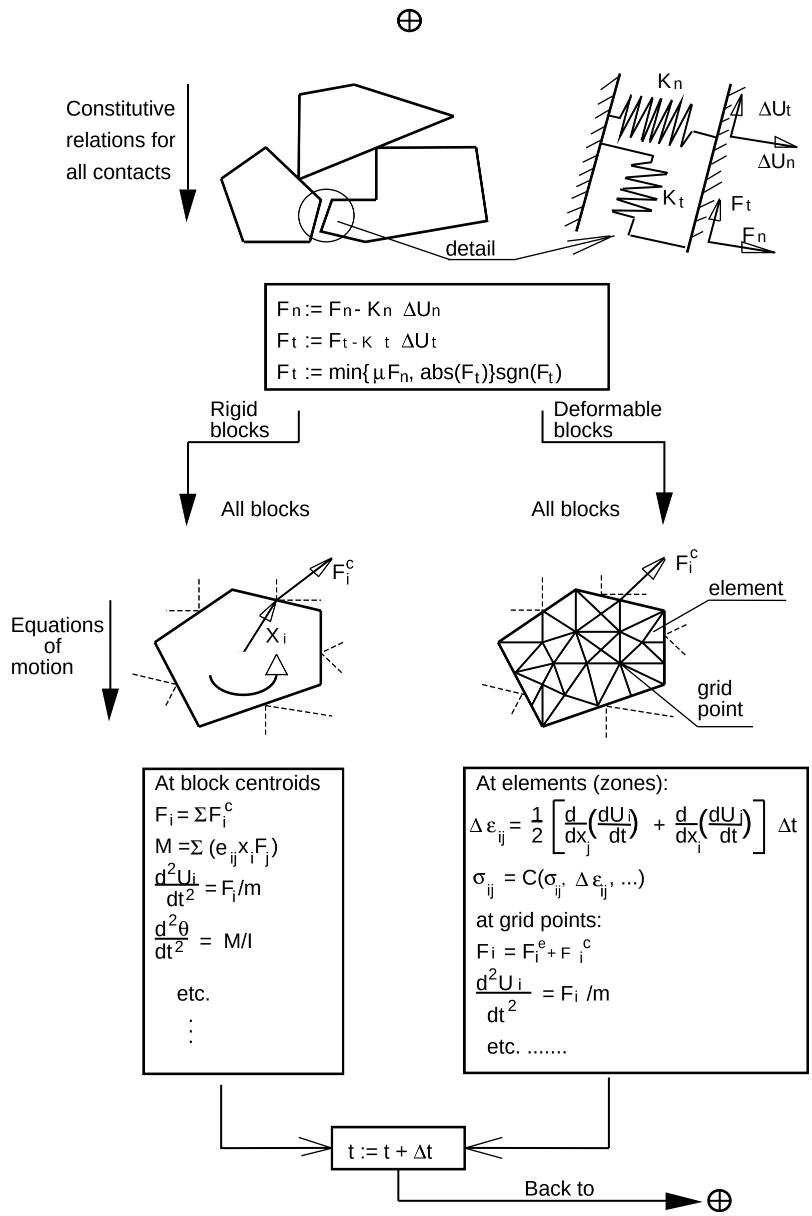

Fig. 17. Calculation cycles in the Distinct Element Method (after Hart, 1993) [247].

- Stress-flow coupling: Makurat et al. (1995) and Liao and Hencher (1997) [277,278];
- Hard rock reinforcement: Lorig (1985) [279];
- Intraplate earthquake: Jing (1990) [280];
- Well and borehole stability: Rawlings et al. (1993) and Santarelli et al. (1992) [281,282];
- Acoustic emission in rock: Hazzard and Young (2000) [283];
- Derivation of equivalent hydro-mechanical properties of fractured rocks: Zhnag et al. (1996), Mas-Ivars et al. (2001), Min et al. (2001) [284–286].

A recent book by Sharma et al. (2001) [287] includes reference to a collection of DEM application papers for various aspects of rock engineering. The applications concentrate on hard rock problems and have increasing focus on coupled hydro-mechanical behaviour—because of the dominating effects of the rock fractures on these aspects, and so where the explicit representation of fractures is necessary. For the softer and weaker rocks, equivalent continuum models are more applicable because there is less difference between the deformability of the fractures and the rock matrix.

Despite the advantages of DEM, lack of knowledge of the geometry of the rock fractures limits its more general applications. In general, the geometry of fracture systems in rock masses cannot be known and can only be roughly estimated. The adequacy of the DEM results in capturing the rock reality are therefore highly dependent on the interpretation of the in situ fracture system geometry—which cannot be even moderately validated in practice. Of course, the same problem applies also to the continuum models, such as the FEM or FDM, but the requirement for explicit fracture geometry representation in the DEM highlights the limitation and makes it more acute. Monte Carlo fracture simulation may help to reduce the level of uncertainty, albeit with increased computation. A prime subject for research, therefore, is increased quality of rock fracture system characterization with more advanced and affordable means, possibly using geophysical exploration techniques.

3.4.2. Implicit DEM—Discontinuous Deformation Analysis method: block systems

DDA originated from a back analysis algorithm for determining a best fit to a deformed configuration of a block system from measured displacements and deformations (Shi and Goodman, 1985) [288]. It was later further developed to perform complete deformation analysis of a block system (Shi, 1988) [231]. The early formulation used a simple representation of block motion and deformation, with six basic variables (three rigid body motion and three constant strain components) and is not suitable for irregularly shaped blocks. The major improvements come from full internal discretization of blocks by triangular or four-noded FEM elements (Shyu, 1993; Chang, 1994) [232,233], as also demonstrated in Jing (1998) [289]. These improvements make the DDA method more suitable for arbitrarily shaped deformable blocks. Improvements of the method have been made for frictional contacts (Jing, 1993) [290], user-friendly code structure and environment (Chen et al., 1996; Chen, 1998) [291,292], rigid block systems (Koo and Chern, 1998) [293], fractured rock masses (Lin et al., 1996) [294] and coupled flow-stress analysis (Kim et al., 2000; Jing et al., 2001) [234,235] with fluid conducted only in fractures. Numerous other extensions and improvements have been implemented over the years in the late 90s', e.g. recently in Doolin and Sitar (2001) [295], with the bulk of the publications appearing in a series of ICADD conferences (Li et al., 1995; Salami and Banks, 1996; Ohnishi, 1997; Amadei, 1999) [81–84].

3.4.2.1. Basic concepts of DDA. By the second law of thermodynamics, a mechanical system under loading (external and/or internal) must move or deform in a direction which produces the minimum total energy of the whole system. For a block system, the total energy consists of the potential energy due to different mechanisms like external loads, block deformation, system constraints, kinetic and strain energy of the blocks and the dissipated irreversible energy. The minimization of the system energy will produce an equation of motion for the block system, the same as that used in the FEM. For a system of N blocks, each having m_i nodes ($i = 1, 2, \ldots, N$), the total number of nodes is $m_1 + m_2 + \cdots + m_N = M$, and each node has two orthogonal displacement variables, u and v. Assuming, without losing generality, that nodes are numbered sequentially blockwise, the minimization will yield ($2M \times 2M$) simultaneous equations, written symbolically as

$$\begin{bmatrix} \mathbf{k}_{11} & \mathbf{k}_{12} & \mathbf{k}_{13} & \ldots & \mathbf{k}_{1N} \\ \mathbf{k}_{21} & \mathbf{k}_{22} & \mathbf{k}_{23} & \ldots & \mathbf{k}_{2N} \\ \mathbf{k}_{31} & \mathbf{k}_{32} & \mathbf{k}_{33} & \ldots & \mathbf{k}_{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{k}_{N1} & \mathbf{k}_{N2} & \mathbf{k}_{N3} & \ldots & \mathbf{k}_{NN} \end{bmatrix} \begin{Bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \mathbf{d}_3 \\ \vdots \\ \mathbf{d}_N \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \vdots \\ \mathbf{f}_N \end{Bmatrix} \quad \text{or} \quad [\mathbf{K}]\{\mathbf{D}\} = \{\mathbf{F}\}, \tag{27}$$

where diagonal sub-matrices $\mathbf{k}_{ij}$ is a ($2m_i \times 2m_i$) matrix representing the sum of contributing sub-matrices for the ith block of m_i nodes. Vector $\mathbf{d}_i$ is a ($2m_i \times 1$) vector of displacement variables of the ith block and vector $\mathbf{f}_i$ is a ($2m_i \times 1$) vector of resultant general forces acting on the ith block. The off-diagonal sub-matrices $\mathbf{k}_{ij}$ ($i \neq j$) represent the sum of contributing sub-matrices of contacts between blocks i and j and other inter-block actions like bolting. The matrix [**K**] can also be called the global "stiffness matrix".

For the three-block system illustrated in Fig. 18, the matrix structure of the equation system by DDA is shown in Fig. 19.

Compared with the explicit approach of the DEM, the DDA method has four basic advantages over the explicit DEM:

(i) The equilibrium condition is automatically satisfied for quasi-static problems without using excessive iteration cycles.
(ii) The length of the time step can be larger, and without inducing numerical instability.
(iii) Closed-form integrations for the element and block stiffness matrices can be performed without the need for Gaussian quadrature techniques.
(iv) It is easy to convert an existing FEM code into a DDA code and include many mature FEM techniques without inheriting the limitations of the ordinary FEM, such as small deformation, continuous material geometry, and reduced efficiency for dynamic analysis. However, matrix equations are produced and need to be solved, using the same FEM technique.

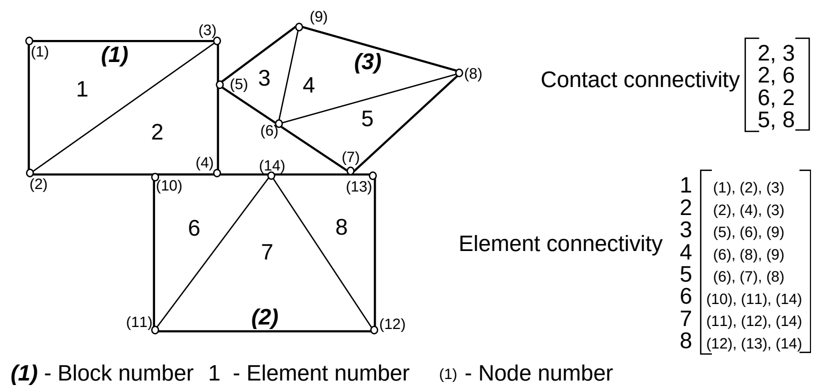

Fig. 18. Blocks and FEM element discretizations by DDA—an example of three blocks.

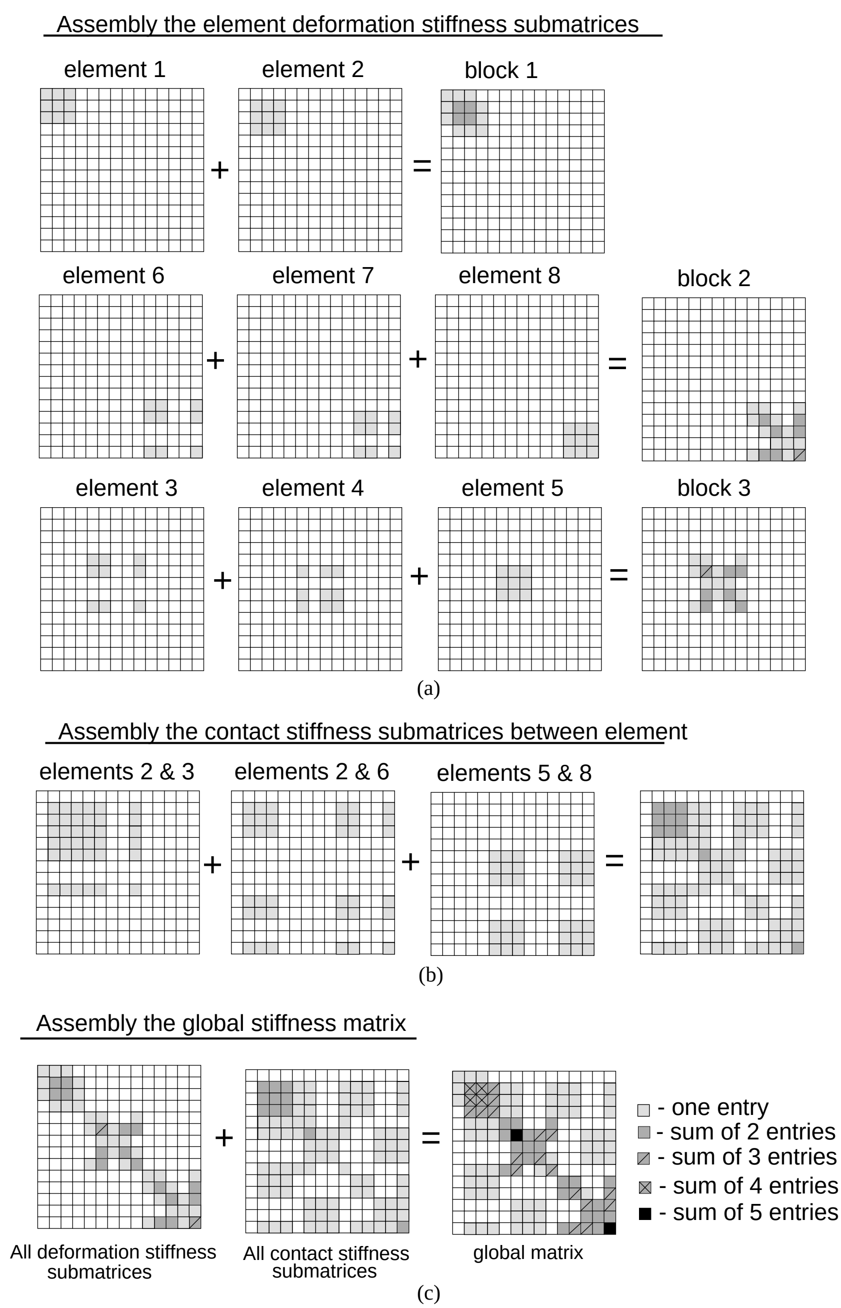

Fig. 19. Matrix structure of the DDA method for the three-block system in Fig. 18.

The DDA method has emerged as an attractive model for geo-mechanical problems because its advantages cannot be replaced by continuum-based methods or explicit DEM formulations. It was also extended to handle 3-D block system analysis (Shi, 2001) [296] and use of higher order elements (Hsiung, 2001) [297], plus more comprehensive representation of the fractures (Zhang and Lu, 1998) [298]. The applications focus mainly on tunnelling, caverns, fracturing and fragmentation processes of geological and structural materials and earthquake effects (see, for example, Yeung and Leong, 1997; Hatzor and Benary, 1998; Ohnishi and Chen, 1999; Pearce et al., 2000; Hsiung and Shi, 2001) [299–303].

3.4.3. Key Block theory

The Key Block approach, initiated independently by Warburton (1983, 1993) [304,305] and Goodman and Shi (1985) [306], with a more rigorous topological treatment of block system geometry in the latter (see also Shi and Goodman, 1989, 1990 [307,308]), is a special method for stability analysis of rock structures dominated by the geometrical characteristics of the rock blocks and hence fracture systems. It does not perform any stress and deformation analysis, but identifies the "key blocks" (or "keystones" in the terms of Warburton (1983) [304]), which are formed by intersecting fractures and excavated free surfaces in rocks and have the potential for sliding and rotation towards certain directions without geometrical constraints.

This technique is a powerful and efficient tool for stability analysis and support design for slopes and underground excavations in fractured rocks—with large-scale movements of isolated blocks (such as wedges in slopes) as the major mode of failure and instability, rather than deformation and stresses of the intact rock matrix. It is therefore suitable for 'hard rock engineering', but is less suitable for soft rocks because the stress/deformation/failure of the rock matrix is equally important for the latter.

Key Block Theory, or simply Block Theory in the Warburton approach, and the associated code development enjoys wide applications in rock engineering, with further development considering Monte Carlo simulations and probabilistic predictions (Stone and Young, 1994; Mauldon, 1993; Hatzor, 1993; Jakubowski and Tajdus, 1995; Kusznaul and Goodman, 1995) [309–313], water effects (Karaca and Goodman, 1993) [314], linear programming (Mauldon et al., 1997) [315], finite block size effect (Windsor, 1995) [316] and secondary blocks (Wibowo, 1997) [317]. Predictably, the major applications are in the field of tunnel and slope stability analysis, such as reported by Chern and Wang (1993), Scott and Kottenstette (1993), Nishigaki and Miki (1995), Yow (1990), Boyle and Vogt (1995), Lee and Song (1998) and Lee and Park (2000) [318–324].

3.4.4. DEM formulations for particle systems

Simulating the mechanical behaviour of granular materials is another important application area of the DEM, for both the Distinct Element and DDA approaches. The principle of the DEM technique for granular materials is basically the same as for the blocks, with the additional simplification that particles are rigid and their shape can be regular (circular, elliptical in 2-D and ellipsoidal in 3-D) or irregular (generally polygonal in 2-D and polyhedral in 3-D). The contacts between the particles are represented by springs, and friction may also be considered.

The seminal work of the DEM for granular materials for geomechanics and civil engineering application is the series of papers by Cundall and Strack (1979a–c, 1982) [325–328], which was based on an earlier work by Cundall et al. (1978) and Strack and Cundall (1978) [329,330]. The development and applications are mostly reported in a series of proceedings of symposia and conferences, such as in Jenkins and Satake (1983), Satake and Jenkins (1988), Biarez and Gourvés (1989), Thornton (1993), Siriwardane and Zaman (1994) in the field of micro-mechanics of granular media in general [331–335], and in Mustoe et al. (1989) and Williams and Mustoe (1993) in the field of geomechanics in particular [336,337]. The simulations of particle systems using the DDA approach appear mostly in ICADD conference systems (Li et al., 1995; Salami and Banks, 1996; Ohnishi, 1997; Amadei, 1999) [81–84].

The most well-known codes in this field are the PFC codes for both 2-D and 3-D problems (Itasca, 1995) [338], and the DMC code by Taylor and Preece (1989, 1990) [339,340]. The method has been widely applied to many different fields such as soil mechanics, the processing industry, non-metal material sciences and defence research. The following examples of publications highlight the wide applications of the DEM for particle systems in the field of rock engineering:

- Fracturing and fragmentation processes of rock blasting: Preece (1990, 1994), Preece and Knudsen (1992), Preece et al. (1993), Preece and Scovira (1994), Donzé et al. (1997), Lee et al. (1997), Lin and Ng (1994) [341–348];
- Ground collapse and movements: Iwashita et al. (1988), Zhai et al. (1997) [349,350];
- Hydraulic fracturing in rocks: Thallak et al. (1991), Huang and Kim (1993), Kim and Yao (1994) [351–353];
- Tunnelling: Kiyama et al. (1991)[354];
- Rock fracture: Blair and Cook (1992) [355].

3.4.5. Dynamic lattice network models

A numerical method closely related to the DEM model of granular material, often called the dynamic lattice network model, has also been applied for simulating fracture initiation and propagation in rocks. The main concept of the lattice model is that the medium consists of a mesh of regular elements, such as triangle elements, with particles of lumped masses located at the vertices of the mesh. The mass particles are then connected by massless springs along the edges of the mesh. The dynamic motion of the medium is simulated by the equations of motions of the mass particles and the deformation of the springs, whose stiffness and strength are derived from those of the medium, which may have local variations and be generated randomly. The masses of the particles are derived from the density of the material, and can also be generated randomly, for representing statistical inhomogeneity of the medium.

This technique is similar to that of the DEM for particle systems, with the difference that it represents the continuum behaviour of the medium through a deterministic/stochastic assembly of particles and springs, rather than as a direct discrete medium.

The applications of the lattice model focus on the fracturing processes of intact rock materials during loading and hydraulic fracturing, such as reported in Paterson (1988), Song and Kim (1994a, 1994b, 1995), Mühlhaus et al. (1997), Schlangen and Van Mier (1994), Li et al. (2000), Napier and Dede (1997) and Place and Mora (2000) [356–364].

3.5. Discrete Fracture Network method

3.5.1. DFN-the basic concepts

The DFN method is a special discrete model that considers fluid flow and transport processes in fractured rock masses through a system of connected fractures. The technique was created in the early 1980s for both 2-D and 3-D problems (Long et al., 1982, 1985; Robinson, 1984; Andersson, 1984; Endo, 1984; Endo et al., 1984; Smith and Schwartz, 1984; Elsworth, 1986a, b; Dershowtz and Einstein, 1987; Andersson and Dverstop, 1987) [365–375], and has been continuously developed afterwards with many applications in civil, environmental and reservoir engineering and other geoscience fields.

The effects of mechanical deformation and heat transfer in a rock mass on fluid flow and transport are difficult to model by the DFN approach and are perhaps crudely approximated without explicit representation of the fractures. Thus, this method is most useful for the study of flow and transport in fractured media in which an equivalent continuum model is difficult to establish, and for the derivation of equivalent continuum flow and transport properties of fractured rocks (Yu et al., 1999; Zimmerman and Bodvasson, 1996) [376,377]. A large number of publications has reported progresses in journals and international symposia and conferences. Systematic presentations and evaluations of the method have also appeared in books, such as Bear et al. (1993), Sahimi (1995), the US National Research Council (1996) and Adler and Thovert (1999) [378–381].

The DFN model is established on the understanding and representation of the two key factors: fracture system geometry and transmissivity of individual fractures. The former is based on stochastic simulations of fracture systems, using the PDFs of fracture parameters (density, orientation, size, aperture or transmissivity) formulated according to field mapping results, in addition to the assumption about fracture shape (circular, elliptical or generally polygonal). Due to the fact that direct mapping can only be conducted at surface exposures of limited area, boreholes of limited length/depth and the walls of underground excavations (tunnels, caverns, shafts, etc.) of more limited exposure areas, and with both lower and upper cut-off limits for mapping, the reliability of fracture network information is dependent on the quality of mapping and representativeness of the sampling, and hence its adequacy is difficult to evaluate. Equally difficult is also the representation of the transmissivity of the fracture population, due to the fact that in situ and laboratory tests can only be performed at a limited number of fracture samples at restricted locations, and the effect of sample scale is difficult to determine.

Despite the above hurdles, the DFN model enjoys wide applications for problems of fractured rocks, perhaps mainly due to the fact that it is a so far, irreplaceable tool for modelling fluid flow and transport phenomena at the 'near-field' scale—the 'near-field' because the dominance of the fracture geometry at small and moderate scales makes the volume averaging principle used in continuum approximations sometimes unacceptable at such scales. Its applicability diminishes for 'far-field' problems at large scales when explicit representation of large numbers of fractures make the computational model less efficient, and the continuum model with equivalent properties become more attractive, similar to the DEM.

There are many different DFN formulations and computer codes, but most notable are the approaches and codes FRACMAN/MAFIC (Dershowitz et al., 1993) [382] and NAPSAC (Stratford et al., 1990; Herbert, 1994, 1996; Wilcock, 1996) [383–386] with many applications for rock engineering projects over the years. Some special features of DFN are briefly reviewed below.

3.5.2. Stochastic simulations of fracture networks

The stochastic simulation of fracture systems is the geometric basis of the DFN approach and plays a crucial role in the performance and reliability of the DFN model, in the same way as the DEM. The key process is to create PDFs of fracture parameters relating to the densities, orientations and sizes, based on field mapping results using borehole logging data and scanline or window mapping techniques, and generate the realizations of the fractures systems according to these PDFs and assumptions about fracture shape (circular discs, ellipses or polygons), (Dershowitz, 1984; Billaux et al., 1989) [387–388].

A critical issue in this technique is the treatment of bias in estimation of the fracture densities and trace lengths from conventional straight scanline or rectangular window mappings. A notable recent development using circular windows (Mauldon, 1998; Mauldon et al., 2001) [389,390] provides an important step forward in this regard.

3.5.3. Solution of the flow fields within fractures

Numerical techniques have been developed for the solution of flow fields for individual fracture elements using closed-form solutions, the finite element model, the boundary element model, the pipe model and the channel lattice model.

Closed-form solutions exist, at present, only for planar, smooth fractures with parallel surfaces of regular shape (i.e. circular or rectangular discs) for steady-state flow (Long, 1983) [391] or for both steady-state and transient flow (Amdei and Illangaseekare, 1992) [392]. For fractures with general shapes, numerical solutions must be used. The FEM discretization technique is perhaps the most well-known techniques used in the DFN flow models and has been used in the DFN codes FRACMAN/MAFIC and NAPSAC. The basic concept is to impose an FEM mesh over the individual discs representing fractures in space (Fig. 20a) and solve the flow equations. The aperture or transmissivity field within the fracture can be either constant or randomly distributed. Similarly, the BEM discretization can also be applied with the boundary elements defined only on the disc boundaries (Fig. 20b), with the fracture intersections treated as internal boundaries in the BEM solution. The compatibility condition is imposed at the intersections of discs. See Elsworth (1986a, b) [372,373] and Robinson (1986) [393] for detailed formulations.

The pipe model represents a fracture as a pipe of equivalent hydraulic conductivity starting at the disc centre and ending at the intersections with other fractures (Fig. 20c), based on the fracture transmissivity, size and shape distributions (Cacas, 1990) [394]. The channel lattice model represents the whole fracture by a network of regular pipe networks (Fig. 3.13d). The pipe model leads to a simpler representation of the fracture system geometry, but may have difficulties to properly represent systems of a number of large fractures.

The channel lattice model is more suitable for simulating the complex flow behaviour inside the fractures, such as the ‘‘channel flow’’ phenomena (Tsang and Tsang, 1987) [395], and is computationally less demanding than the FEM and BEM models since the solutions of the flow fields through the pipe elements are analytical.

The fractal concept has also been applied to the DFN approach in order to consider the scale dependence of the fracture system geometry and for upscaling the permeability properties, using usually the full box dimensions or the Cantor dust model (Barton and Larsen, 1985; Chilés, 1988; Barton, 1992) [396–398]. Power law relations have been also found to exist for trace lengths of fractures and have been applied for representing fracture system connectivity (Renshaw, 1999) [399].

3.5.4. Issues of importance and difficulty

The influence of the rock matrix on flow in rock fractures is usually not considered in the DFN models. However, the related effects also need to be estimated

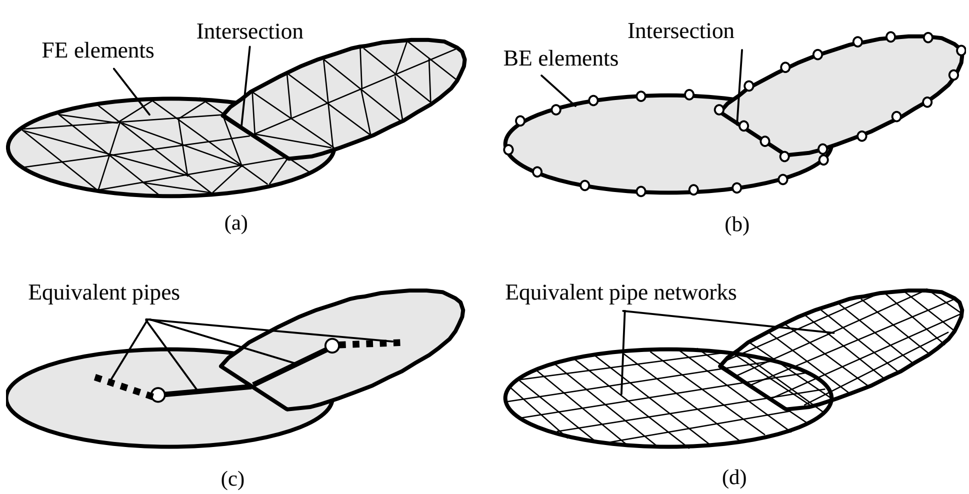

Fig. 20. Representation of rock fractures for the flow equation solution: (a) FEM; (b) BEM; (c) equivalent pipes; and (d) channel lattice model.

when the permeability of the rock matrix is large compared with that of the fractures, or the time scale of the problem is long enough so that matrix diffusion cannot be ignored. In such cases, a fracture system embedded in a highly porous matrix needs to be properly represented, considering the different time scales between the fracture flow and matrix diffusion processes, such as the FEM technique used by Sudicky and McLaren (1992) [400]. Dershowitz and Miler (1995) [401] reported a simplified technique for considering matrix influence on flow in DFN models, using a probabilistic particle tracking technique.

The stress/deformation processes of the fractures, and the effects of stress/deformation of rock matrix on the deformation/flow of fractures, are usually also ignored in the DFN models, simply due to the complexity of the numerical techniques and extraordinary computational efforts needed. Although simple estimates concerning the effects of in situ stresses on fracture aperture variations have been used in DFN models, e.g. in the NAPSAC code (Wilcock, 1996) [386], proper representation of the coupled stress/flow process in fractures is needed, especially for near-field problems for performance and safety assessment of radioactive waste repositories.

Simulation of multiphase flow and transport by DFN models is also an important subject, with significance for reservoir simulations (for oil/gas recovery and hot-dry-rock thermal energy extraction projects) and nuclear waster repositories (heat decay and waster phase change, and radionuclide transport). The conventional technique is the dual permeability models which treats the fracture system and matrix as two superimposed conducting media for flow, heat transfer and transport, because the heat transfer process is largely dominated by matrix heat conduction—but heat convection by flow in fractures may also become important issues, e.g. for hot-dry-rock reservoir simulations and performance of the near-field buffer material for nuclear waste isolation (Pruess and Wang, 1987; Slough et al., 1999; Hughes and Blunt, 2001) [402–404]. Proper solutions of the governing equations for mass and energy balances, with phase change relations (water evaporation and condensation, moisture flow driven by thermal gradients, etc.) need to be properly considered, with embedded DFN models in porous media, which is a subject still needed to be further investigated. The main challenge is the computational effort for considering the matrix–fracture interaction, with fractures dominating flow and matrix dominating heat transfer. Phase changes may occur in both.

Because of its origin based on fracture mapping at limited exposure areas, the DFN models are based on, and at the same time are limited by, their two cornerstones: the largely unknown true fracture geometry; and the hydraulic properties of fractures—with, in both cases, the attendant difficulties in property measurement and evaluation. The upper and lower cut-off limits for trace lengths in mapping, and the effects of scale, roughness, and stress-dependency of fracture aperture and transmissivity, play a significant role in the adequacy and reliability of the DFN models: it is often difficult to evaluate their significance because of the lack of any independent checking mechanisms. An additional drawback is the high computational demands when large fracture numbers are required for large-scale problems.

These drawbacks are similar to those of the DEM models and can only be partially overcome or qualitatively addressed with the aid of advanced mapping and measurement techniques. Therefore, the applications of DFN models are concentrated more on characterization of the permeability of fractured rocks and generic studies of fracture influences, and the design of rock engineering works for near-field problems, see for example as follows:

- Hot-dry-rock reservoir simulations: Layton et al. (1992), Ezzedine and de Marsily (1993), Watanabe and Takahashi (1995), Kolditz (1995), Willis-Richards and Wallroth (1995), Willis-Richards (1995) [405–410];
- Characterization of permeability of fractured rocks: Dershowitz et al. (1992), Herbert and Layton (1995), Doe and Wallmann (1995), Barthélémy et al. (1996), Jing and Stephansson (1996), Margolin et al. (1998), Mazurek et al. (1998), Zhang and Sanderson (1999) [411–418];
- Water effects on underground excavations and rock slopes: Rouleau and Gale (1987), Xu and Cojean (1990), He (1997) [419–421].

3.5.5. Alternative formulations—percolation theory

Percolation theory is a counterpart of the lattice model for solid deformation and fracturing for fluid diffusion process. The theory is based on a random lattice model of conductors (fractures) for deriving fluid transport conditions (permeability), based on the connectivity through a geometrical sample of the fractures (Robinson, 1984; Hestir and Long, 1990; Berkowitz and Balberg, 1993; Sahimi, 1995) [367,422,423,379]. The theory provides a theoretical platform for understanding the geometric conditions for fluid conduction in fractured media in terms of a percolation threshold, expressed as a critical probability. Application of this theory in rock mechanics and engineering problems seems to concentrate on characterization of flow properties of fractured rocks. Some example works in this field are given bellow:

- Critical behaviour of deformation and permeability of fractured rocks: Zhang and Sanderson (1998) [424];

- Flow and transport in fracture networks: Mo et al. (1998) [425];
- Microstructure and physical properties of rock: Guéquen et al. (1997) [426];
- Fluid, heat and mass transport in percolation clusters: Kimmich et al. (2001) [427].

3.6. Hybrid models

Hybrid models are frequently used in rock engineering, basically for flow and stress/deformation problems of fractured rocks. The main types of hybrid models are the hybrid BEM/FEM, DEM/BEM models. The hybrid DEM/FEM models are also developed. The BEM is most commonly used for simulating far-field rocks as an equivalent elastic continuum, and the FEM and DEM for the non-linear or fractured near-field where explicit representation of fractures or non-linear mechanical behaviour, such as plasticity, is needed. This harmonizes the geometry of the required problem resolution with the numerical techniques available, thus providing an effective representation of the effects of the far-field to the near-field rocks.

3.6.1. Hybrid FEM/BEM models

The hybrid FEM/BEM was first proposed in Zienkiewicz et al. (1977) [428], then followed by Brady and Wassyng (1981) [429], and Beer (1983) [430] as a general stress analysis technique. In rock mechanics, it has been used mainly for simulating the mechanical behaviour of underground excavations, as reported in Varadarajan et al. (1985), Ohkami et al. (1985), Gioda and Carini (1985), Swoboda et al. (1987) and van Estorff and Firuziaan (2000) [431–435]. The coupling algorithms are also presented in detail in Beer and Watson (1992) [49].

The standard technique is to treat the BEM region as a 'super' element with an artificially 'symmetrized' stiffness matrix, using the least-square techniques, so that it can be easily inserted into the symmetric FEM stiffness matrix for the final solution, which is easier to handle than the non-symmetric BEM stiffness matrix. However, such artificial 'symmetrization' introduces additional errors into the final system equations. The coupling can also be performed in the opposite direction, i.e. treat the FEM region as a 'super' BEM element, and insert the corresponding FEM stiffness matrix into the final BEM stiffness matrix; this leads to a asymmetric stiffness matrix for the final equation, which needs additional computational efforts for solution.

The hybrid BEM/FEM models are as efficient computationally as the FEM, with the additional advantage of being able to deal with the non-linear behaviour of materials in the FEM region, using the FEM's advantages. However, this advantage may be affected by the symmetrized BEM equation. A possible step forward in this direction is to use the Galerkin double integration techniques in the BEM region so that the final BEM stiffness matrix is automatically symmetric, and therefore can be directly inserted in the final hybrid BEM/FEM matrix without errors caused by artificial "symmetrization".

3.6.2. Hybrid DEM/BEM

The hybrid DEM/BEM model was implemented only for the explicit Distinct Element Method, in the code group of UDEC and 3DEC. The technique was created by Lorig and Brady (1982, 1984, 1986) [436,3,437], and was implemented into UDEC by Lemos (1987) [244]. The basic concept is to treat the BEM region (which surrounds the DEM region) as a 'super' block having contacts with smaller blocks along the interfaces with the DEM region (cf. Fig. 5), which can be treated in standard DEM contact representations. The key conditions are:

(1) the kinematic continuity along the interfaces of the two regions during the time-marching process; and
(2) the elastic properties of the two regions near the interface are similar.

Condition 2 indicates that blocks in the DEM regions must be deformable, i.e. not be rigid blocks. In the case of mixed rigid and deformable block systems, special equations of motion need to be developed to handle such cases.

Wei (1992) and Wei and Hudson (1998) [438,439] reported a development of hybrid discrete-continuum models for coupled hydro-mechanical analysis of fractured rocks, using combinations of DEM, DFN and BEM approaches. The near-field of a fractured rock mass is simulated using DEM and DFN models, using independent DFN and DEM codes, for representing the dominance of fractures on the near-field fluid flow and stress/deformation of rock blocks, and the far-field is simulated by BEM codes for flow and stress/deformation in a continuum. The equations of flow and motion are not directly coupled, but are solved independently by separate DFN, DEM and BEM codes, and they are coupled through an internal linking algorithm with the time-marching process.

3.6.3. Alternative hybrid models

Besides the above mainstream hybrid formulations, there are other coupling techniques which take advantage of different numerical methods. Pan and Reed (1991) [440] reported a hybrid DEM/FEM model, in which the DEM region consists of rigid blocks and the FEM region can have non-linear material behaviour. The algorithm places the FEM calculations into the DEM time-marching process. Since the blocks in DEM region are rigid and the FEM region is an elastic continuum, the kinematical continuity condition along the interface of the DEM and FEM regions may not be satisfied.

A hybrid beam-BEM model was reported by Pöttler and Swoboda (1986) [441] to simulate the support behaviour of underground openings, using the same principle as the hybrid BEM/FEM model. Sugawara et al. (1988) [442] reported a hybrid BEM-characteristics method for non-linear analysis of rock cavern.

3.7. Neural networks/empirical techniques

All the numerical modelling methods described so far are in the category of '1:1 mapping'—using the terms in Fig. 2, Section 1, which illustrates the eight basic methods of rock mechanics modelling and rock engineering design. The term '1:1 mapping' refers to the attempt to model geometry and physical mechanisms directly, either specifically or through equivalent properties.

A completely different numerical method, located in Box 2C in Fig. 2, uses neural networks: a 'non-1:1 mapping' method. The rock mass is represented indirectly by a system of connected nodes, but there is not necessarily any physical interpretation of the nodes, nor of their input and output values. The model purports to operate like the human brain.

Another example is the use of empirical techniques, the rock rating systems (Q, RMR, RMI, GSI, etc.—see Box 2B in Fig. 2), for characterization of rock mass properties and construction design, without resorting to the solution of the basic balance equations and use of thermodynamically acceptable constitutive models. This design method is possible because of the fact that a true 1:1 mapping solution can never be achieved with 100% confidence and reliability for problems in fractured rocks because a 100% characterization of the rock/fracture system can never be achieved and validated. This is the reason for the development and success of non-1:1 mapping techniques like the rating systems in rock engineering. Through the use of empirical techniques, including empirical failure criteria, a rock engineer can state whether a tunnel or a cavern is safe or dangerous, without solving any of the equations in the BEM, FEM or DEM methods.

Such a 'non-1:1 mapping' system has its advantages and disadvantages. The advantages are:

- that the geometrical and physical constraints of the problem, which appear in governing equations and constitutive laws when the 1:1 mapping techniques are used, are no longer so dominating,
- different kinds of neural networks and empirical models can be applied to a problem, and
- there is the possibility that the 'perception' we enjoy in the human brain may be mimicked in the neural network, so that the programs can incorporate judgements based on empirical methods and experiences.

The disadvantages are that

- the procedure may be regarded as simply super-complicated curve fitting (because the program has to be 'taught'),
- the model cannot reliably estimate outside its range of training parameters,
- critical mechanisms might be omitted in the model training, and
- there is a lack of theoretical basis for verification and validation of the techniques and their outcomes.

Neural network models provide descriptive and predictive capabilities and, for this reason, have been applied through the range of rock parameter identification and engineering activities. Recent published works on the application of neural networks to rock mechanics and rock engineering includes the following:

- Stress–strain curve for intact rock: Millar and Clarici (1994) [443];
- Intact rock strength: Alvarez Grima and Babuska (1999); Singh et al. (2001) [444,445];
- Fracture aperture: Kacewicz (1994) [446];
- Shear behaviour of fractures: Lessard and Hadjigeorgiou (1999) [447];
- Rock fracture analysis: Sirat and Talbot (2001) [448];
- Rock mass properties: Qiao et al. (2000); Feng et al. (2000) [449,450];
- Rock mechanics models: Feng and Seto (1999) [451];
- Rock mass classification: Sklavounos and Sakellariou (1995); Liu and Wang (1999) [452,453];
- Displacements of rock slopes: Deng and Lee (2001) [454];
- Tunnel boring machine performance: Alvarez Grima et al. (2000) [455];
- Displacements/failure in tunnels: Sellner and Steindorfer (2000); Lee and Sterling (1992) [456,457];
- Tunnel support: Leu (2001); Leu et al. (2001) [458,459];
- Surface settlement due to tunnelling: Kim et al. (2001) [460]
- Earthquake information analysis: Feng et al. (1997) [461];
- Rock engineering systems (RES) modelling: Millar and Hudson (1994); Yang and Zhang (1998) [462,463];
- Rock engineering: Yi and Wanstedt (1998) [464];
- Overview of the subject: Hudson and Hudson (1997) [465].

As evidenced by the list of highlighted references above, the neural network modelling approach has already been applied to the variety of subjects in rock mechanics and rock engineering. It is also evident that the method has significant potential—because of its 'non-1:1 mapping' character and because it may be possible in the future for such networks to include creative ability, perception and judgement. However, the method has not yet provided an alternative to conventional modelling, and it may be a long time before it can be used in the comprehensive Box 2D mode envisaged in Fig. 2 and described in Feng and Hudson (2003) [466].

3.8. Constitutive models of rocks

The constitutive models of rocks, including those for both rock fractures and fractured rock masses, are one of the most important components of numerical solutions for practical rock engineering problems, and one of the most intensively and continuously investigated subjects in rock mechanics. The most recent developments in the area are briefly introduced with some comments, supported by literature sources. To make the presentation clearer, the models are divided into five groups according to their different formulation platforms and traditional application areas: classical constitutive models, failure criteria, time effects and viscosity, size effects and homogenization, damage mechanics models, and rock fracture models.

3.8.1. Classical constitutive models of rocks

The classical constitutive models are the models based mostly on the theory of elasticity and plasticity, but with special considerations of fracture effects. The model of linear elasticity based on the generalized Hooke's law is still by far the most widely adopted assumption for the mechanical behaviour of rocks, especially for hard rocks. When the CHILE assumption (Continuous, Homogeneous, Isotropic, Linear Elastic) is adopted, the constitutive law is simply characterized by two independent material properties, either the most commonly known, Young's modulus (E) and Poisson's ratio (v), or Lame's two parameters, G and λ. More sophisticated constitutive models of anisotropic elasticity can be derived in closed-form by considering alternative elastic symmetry conditions for intact rocks (such as transversely isotropic elasticity) or equivalent or effective continuum elastic rocks intersected by orthogonal sets of infinitely large or finite fractures, see Singh (1973), Gerrard (1982), Fossum (1985), Wei and Hudson (1986), Yashinaka and Yamabe (1986), Wu (1988), Murakami and Hegemier (1989), Chen (1989), Singh (2000), and Wu and Wang (2001) [467–476]. Oda (1986) [477] developed a crack tensor approach to derive the equivalent elastic compliance tensors of fractured rocks with randomly distributed finite fractures, and Li et al. (1998) developed effective stress–strain relations for rocks containing deformable micro-cracks under compression. In the above developments, interactions between the fractures were not considered. A comprehensive summary of the constitutive laws for geomaterials in oil and gas reservoir rocks is given by Papamichos (1999) [478].

Plasticity and elasto-plasticity models have been developed and widely applied to fractured rocks since the 1970s, based mainly on the classical theory of plasticity, with typical models using Mohr–Coulomb and Hoek–Brown failure criteria (Hoek, 1983; Hoek and Brown, 1982, 1997) [479,132,480] as the yield functions and plastic potentials. A comprehensive and detailed description of such plasticity models is given in Owen and Hinton (1980) [45], together with FEM implementations. Parallel and similar developments can also be seen in Zheng et al. (1986) [481] with a strain space formulation, Adhikary and Dyskin (1998) [482] for layered rocks, Sulem et al. (1999) [483] for elasto-plasticity models of a sandstone and Boulon and Alachaher (1995) [484] for a non-linear model based on generalized strain paths, suitable for FEM implementations.

Strain-hardening and strain-softening are the two main features of plastic behaviour of rocks, with the latter more often observed under uniaxial compression test conditions. Related works are reported by Dragon and Mróz (1979), Nemat-Nasser (1983), Zienkiewicz and Mróz (1984), Read and Hegemier (1984), Gerrard and Pande (1985), Desai and Salami (1987), Rowshandel and Nemat-Nasser (1987), Kim and Lade (1988), and Sterpi (1999) [485–493].

Strain-localization (e.g. shear-banding) is a deformation phenomenon closely related to the constitutive models of rocks, and has been studied intensively using plasticity and damage mechanics models (see, for example, Rudnicki and Rice, 1975; Fang, 2001) [494,21].

3.8.2. Failure criteria

The failure criteria of rocks are important components of constitutive relations and are usually used as yield surfaces or/and plastic potential functions in a plasticity model. Besides the most well known and perhaps also the most widely used Mohr–Coulomb and Hoek–Brown criteria, different failure (or strength) criteria have been proposed for rock masses over the years. The dimensionless forms of the Mohr-type failure criteria were discussed in Pariseau (1994) [495]. Sheorey (1997), Mostyn and Douglas (2000) and Parry (2000) have provided a comprehensive reviews of the subject [496–498]. Some of the recent developments are listed below with their contexts:

- Anisotropy of jointed rock mass strength: Amadei and Savage (1989) [499];
- Time-dependent tensile strength of saturated granite: Sun and Hu (1997) [500];
- Failure criteria for anisotropic rocks: Duvean et al. (1998) [501];
- Compressive failure of rocks: Gupta and Bergström (1997), Tharp (1997) [502,503];
- Testing factors for establishing rock strength: Hawkins (1998) [504];
- Shear failure envelope for modified Hoek–Brown criterion: Kumar (1998) [505];
- Effect of intermediate principal stress on strength of anisotropic rocks: Singh et al. (1998) [506];
- Modified Mohr–Coulomb failure criterion for layered rocks: Lai et al. (1999) [507];
- Short- and long-term strength of isotropic rocks: Aubertin et al. (2000) [508];
- Relation between failure criterion and deformability of rock: Hibino et al. (2000) [509];
- Water effects on rock strength: Masuda (2001) [510];
- Failure criterion for transversely isotropic rocks: Tien and Kuo (2001) [511].

3.8.3. *Time effects and viscosity*

Time effects are one of the most important and also perhaps one of the least understood aspects of the physical behaviour of rock masses. There are two main aspects: the effects caused by the rock (and fracture) viscosity and the effects caused by the dynamic loading conditions. The former concerns mainly the stationary behaviour of rock over long- or extremely long-terms, such as geological time periods, and the latter is just the opposite—dynamic and even violent behaviour over short durations, such as earthquake effects. In such cases, not only the magnitudes, directions and durations, but also the rate of change in loading parameters are important. The time and rate effects are therefore often discussed in combination.

The effect of viscosity, also termed a rock rheology effect, is significant in rock salt and other weak rocks, and is caused by two mechanisms: creep and relaxation. The former is the behaviour of increasing deformation (strain) under constant loading (stress); and the latter is the decreasing loading (stress) while the deformation (strain) state is kept constant. The fundamentals of the physics, experimental basis and the constitutive models for these two mechanisms of viscosity were described in detail in Jaeger and Cook (1969) [512] and Cristesu and Hunsche (1998) [513], with applications in tunnel/cavern failure and borehole closure in mining and petroleum engineering.

The effect of viscosity has been considered in constitutive modelling in combination with other basic deformation mechanisms, such as elasticity and elasto-plasticity or plasticity (leading to constitutive models of so-called visco-elasticity, and its plasticity counterparts, visco-elasto-plasticity/visco-plasticity). Comprehensive descriptions of the constitutive models using visco-plasticity were given in Valanis (1976) [514] and in Owen and Hinton (1980) [45]. The link to and coupling with fluid flow are also important. Representative and recent works are listed below:

- Parameter identification of visco-elastic materials using back-analysis: Ohkami and Ihcikawa (1997), Ohkami and Swoboda (1999), Yang et al. (2001) [515–517];
- Viscosity and yield strength degradation of rock: Nawrocki and Mróz (1999) [518];
- Visco-elasto-plastic behaviour with finite strains: Nedijar (2002a, b) [519,520];
- Comparison of formats and algorithms of visco-plasticity models: Runesson et al. (1999) [521];
- Combination with fluid flow and rock porosity: Abousleiman et al. (1993, 1996), [522–523];
- Material softening simulations: Diez et al. (2000) [524];
- Rheology of fractured rocks: Patton and Fletcher (1998) [525];
- Poroelasticity of rocks with anisotropic damage: Shao et al. (1997) [526].

3.8.4. *Size effects and homogenization*

Size effect is a special feature of fractured rocks, mainly due to two factors caused by the existence of fractures of various sizes in rock masses. The first is the fact that the fracture systems divide the rock mass into a large number of sub-domains or blocks, whose sizes and interactions dominant the overall behaviour of rock masses. The second is the fact that the physical behaviour of fractures themselves is dependent on the sizes of fractures, due to scale-dependence of surface roughness of the rock fractures (Fardin et al., 2001) [527]. The state-of-the-art of this subject for rocks is presented in two edited volumes by Da Cunha (1990, 1993) [4,5] and a recent comprehensive survey of size effects in the strength and behaviour of structures, including geotechnical structures, was given by Bažant (2000) [528].

A shortcoming of classical plasticity theory is the lack of an intrinsic length scale in the models that is otherwise needed to explain the size effect in such theories. Since the 1990s, however, a theory of gradient plasticity was developed, which may be applied to consider plastic behaviour and Strain-localization of geological materials with regular fracture patterns, such as Aifantis (1992), Zbib and Aifantis (1989, 1992) [529–531]. Frantziskonis et al. (2001) [532] proposed a new scale-dependent constitutive model for heterogeneous materials like concretes, using wavelet analysis techniques. Application of this theory has not been found in rock mechanics publications.

The most representative and contemporary methods to take the size effect into account for the physical behaviour of fractured rocks is the equivalent continuum approach established on the basis of the Representative Elementary Volume (REV), through analytical or numerical processes of homogenization and/or upscaling and based on assumed constitutive models. The properties established are often called effective or equivalent properties. Except for a few cases with specific fracture sets (often assumed to be infinitely large in size), closed-form solutions for equivalent properties do not exist for generally fractured rocks, and numerical simulations are often used as the tools of derivation. One exception is the crack tensor theory proposed by Oda (1986) [477], in which random fracture populations can be readily incorporated into the analytical functions defining the compliance and permeability tensors.

For homogenization and upscaling, much work has been undertaken in general solid mechanics fields, especially for composite materials. The general methodology is well known, see for example the most recent publication by Fraldi and Guarracino (2001) [533]. For the mechanical properties of fractured rocks, a number of closed-form solutions were obtained with simplified fracture system geometry, as mentioned in Section 3.8.1.

Homogenization and upscaling techniques have a long history of application in deriving equivalent hydraulic properties of fractured rocks, such as in Snow (1965) [534]. Two comprehensive reviews by Renard and de Marsily (1997) [535] and Wen and Gómez-Hernández (1996) [536] summarized the state-of-the-art. The more recent works on the subject are reported in Lee et al. (1995), Li et al. (1995), Tran (1996), Hristopulos and Christakos (1997), Scheibe and Yabusaki (1998), Pozdniakov and Tsang (1999), Shahimi and Mehrabi (1999), Lunati et al. (2001) and Zhang and Sanderson (1999) [537–544,418]. Derivation of coupled hydro-mechanical effective properties of general porous media was reported in Kachanov et al. (2001) [545]. Similar works were also reported for heat transfer processes in porous media, see Quintard et al. (1997) [546].

For flow in fractured porous media like rock masses, new techniques were also created to separate the contributions from the rock matrix and from the fracture systems (which are also treated as equivalent continua), the so-called dual (double) porosity models, dual (double) permeability models and dual (double) continuum models, in order to simplify the complexity in the fracture–matrix interaction behaviour and to partially consider the size effects caused mainly by the fractures. Some recent works in this field can be seen in Zimmerman et al. (1996), Bai (1997), Bai et al. (1999), Choi et al. (1997), Masters et al. (2000), McLaren et al. (2000), Vogel et al. (2000), Zhang et al. (2000), and Landereau et al. (2001) [547–555]. In these approaches, although the matrix and fractures are numerically 'separated', the size effects in the fracture system still exist, and the interactions between the matrix continua and fracture 'continua' still need to be considered. It is more than likely that the two 'continua' will have different REVs. Proper numerical methods to treat the effect of such differences in size effects of co-existing continua remain to be developed.

In the above approaches and models, the interactions between the finite fractures were not considered. It appears that an effective approach to derive equivalent elastic constitutive models of rocks with randomly distributed finite fractures is the numerical approaches of homogenization and upscaling, such as reported by Stietel et al. (1996) and Lee and Pande (1999) using FEM [556,557], and Mas-Ivas et al. (2001) and Min et al. (2001) using DEM [285,286].

The existence of the REV for fractured rocks is still in debate and non-REV approaches were also proposed in Pariseau (1999) [558]. The focus of debate is whether such REVs can exist physically considering the presence of a hierarchic structure of fracture sizes and widths (apertures) and their vastly different physical behaviour and properties.

3.8.5. Damage mechanics models

Constitutive models of rocks have also been developed using continuum damage mechanics principles, proposed first by Kachanov (1958) [559], based on scalar, vector or tensor representations of the void formation, micro-cracking or embedded fracture phenomena in rock under loading. This theory is very closely related to both continuum mechanics and fracture mechanics, and serves as a bridge connecting the two (see Oliver, 2000; Oliver et al., 2002) [560,561]. It has a certain parallelism in the formulation with the plasticity models, such as using damage evolution laws in place of flow rules. The restriction by the normality rule in plasticity theory is, however, absent in damage mechanics principles. The damage mechanics theory has also a certain advantage in simulating the Strain-localization factors using continuum approaches and study of brittle-ductile deformation model transitions observed during testing rock samples. Comprehensive reviews on its development, characteristics, trends and weaknesses are given in Krajcinovic (2000) and de Borst (2002) [562,563].

The damage mechanics approach has been applied to study strength degradation and Strain-localization phenomena in rocks and to formulate damage related constitutive models of rock and rock like materials, such as Kawamoto et al. (1988) and Ichikawa et al. (1990) [564,565]. A large number of papers has been published on these subjects and cannot be summarized in even moderate detail. Below are some recent publications concerning rock damage:

- Constitutive models and properties of rocks: Aubertin and Simon (1997), Grabinsky and Kamaleddine (1997), Shao (1998), Shao and Rudnicki (2000), Yang and Daemen (1997), Chazallon and Hicher (1998), Homand-Etienne et al. (1998), Basista and Gross (1998), Zhao (1998), Carmelier (1999), Chen (1999), Dragon et al. (2000), Jessell et al. (2001), Li et al. (2001), Brencich and Gambarotta (2001) [566–580];
- Damage induced permeability change in rock: Souley et al. (2001) [581];
- Strain-localization and failure predictions: Comi (1999, 2001), Peerlings et al. (2002) [582–584];
- Scale effects of damage in cemented granular rocks: Pisarenko and Gland (2001) [585];
- Effects of damage on explosions, blasting and fragmentation of rocks: Yang et al. (1996), Liu and Katsabanis (1997a, b), Rossmanith and Uenishi (1997), Ma et al. (1998), Wu et al. (1999) [586–590];
- Disturbed state concept-based model: Desai and Zhang (1998) [591];
- Effects of damage on excavations and boreholes: Sellers and Scheele (1996), Cerrolaza and Garcia (1997), Elata (1997); Nazimko et al. (1997a, b), Swoboda et al. (1998), Zhu and Li (2000), Zhu et al. (1999) [592–598,273].

3.8.6. Rock fracture models

Constitutive models for rock fractures play an important role in almost every aspect of rock mechanics and rock engineering, especially in the fields of numerical modelling and characterization. It is therefore not surprising that the mechanics and models of rock fractures has become one of the main themes in almost every international or national conference on rock mechanics, rock physics and rock engineering. The most directly relevant volumes are those edited by Stephansson (1985), Barton and Stephansson (1990), Myer et al. (1995) and Rossmanith (1990, 1995, 1998) [599–604]. The subject has also become an inevitable part of many text and reference books and parts of many edited volumes, such as Chernyshev and Dearman (1991), Lee and Farmer (1993), Selvadurai and Boulon (1995), Hudson (1993), Indrarantna and Hague (2000) and Indrarantna and Ranjith (2001) [605–610]. Some early comprehensive reviews on the experimental aspects, formulations of constitutive models and strength envelopes of rock fractures are given in Jing and Stephansson (1995), Ohnishi et al. (1996) and Maksimovic (1996), respectively [611–613]. In this section, we will only present a limited number of fundamental developments of importance.

The constitutive models of rock fractures are formulated with mainly two approaches: empirical and theoretical. The primary variables are contact tractions and relative displacements (instead of stresses and strains in continuum models), and aperture and flow rates (fluxes). Implementation of constitutive models into continuum numerical methods such as FEM often leads to so-called interface elements, which may also cause numerical instabilities when zero-thickness of interface elements is employed, such as discussed in Kaliakin and Li (1995), Day and Potts (1994), and Lee and Pande (1999) [614–615,557]. Implementation into DEMs is generally a more straightforward use of contact mechanics principles, but the prevention of inter-penetration of solid blocks must be applied, using methods like penalty function, Lagrangian multipliers or augmented Lagrangian multiplier techniques.

Besides the classical Coulomb or Mohr–Coulomb friction laws, the most well-known constitutive laws developed for rock fractures with an empirical approach are Goodman's model (Goodman et al., 1968; Goodman, 1976) [51,52] concerning mechanical behaviour and the Barton–Bandis model concerning coupled hydro-mechanical responses (Bandis et al., 1983; Barton et al., 1985) [616,617]. Since the principles of the thermodynamics of solids are not involved in the formulations, such empirical models may possibly violate the second law of thermodynamics when complex stress-displacement paths are involved. In addition, the validity of such models beyond their model construction databases may or may not be verifiable. On the other hand, such empirical models provides basic understanding to the physical behaviour of the fractures and are still useful in engineering practice due to their usually simpler mathematical forms and reduced number of parameters required, especially when simple loading mechanisms are anticipated. There are many other empirical models of rock fractures implemented in numerous computer codes, such as the Continuously Yielding model in UDEC (ITASCA, 1992) [212], micro-mechanics based models (Dong and Pan, 1996) [618] and the DSC (disturbed state model)-based models (Desai and Ma, 1992; Desai, 1994) [619,620], but the majority of such models used in practice are the above three empirical models or their different variations.

The theoretical models are formulated using principles of one of the solid mechanics branches, mostly plasticity or contact mechanics. Consistency with the thermodynamic principles is strictly required in such models and therefore theoretical models are established on the firmer basis of solid physics compared with empirical models. However, such theoretical correctness is achieved at the expense of usually more complex mathematical forms and an increased number of parameters, causing extra considerations and costs of parameterization when using such models.

Besides the constitutive model of rock fracture proposed by Amadei and Saeb (1990) [621], which has been extended by Souley et al. (1995) [622], the most common constitutive models for rock fractures developed with the theoretical approach are formulated using principles of plasticity theory, based on the similarity between the plastic hardening–softening deformation of solids and shear stress (traction)-shear displacement components of rock fractures. Examples of early work in this direction can be seen in Ghaboussi and Wilson (1973) [53] and Fishman and Desai (1987) [623]. Plesha (1987) [624] proposed a plasticity-based model for rock fractures, using a stress-transformation function based on a simplified asperity model relating the micro- and macro-contact stresses on the fracture surfaces, and an exponential degradation law of the asperity angle based on the dissipated work accumulated during shearing process. This work has inspired many similar developments, such as Jing et al. (1993, 1994) [274,275], Nguyen and Selvadurai (1998), Lee et al. (2001), Plesha and Ni (2001) and Lee and Cho (2002) [625–628]. Different models using plasticity theory as the formulation platform were also reported by Buczkowski and Kleiber (1997) [60], Desai and Fishman (1991), Desai et al. (1995), Mróz and Giambanco (1996) and Lespinasse and Sausse (2000) [629–632].

The principles of contact mechanics of rough surfaces has also been used to formulate constitutive models of rock fractures, such as the early work by Swan (1983) and Sun et al. (1985), Swan and Sun (1985), and later by Yoshioka and Scholz (1989a, b) and Lei et al. (1995) [633–638]. The work is based mainly on principles established in Greenwood and Williamson (1966) [639] and Greenwood and Tripp (1971) [640] simulating the contacts, friction and wear of rough surfaces. The practical applicability of such models depends largely on the unique quantification of surface roughness and understanding its impact on fracture behaviour, which still remain as a challenging topic today, and an ever-continuing research subject for often the non-stationary roughness of rock fractures. The concept of Joint Roughness Coefficient (JRC) (Barton, 1973; Barton and Choubey, 1976) [641,642] and other random field and geostatistical models, especially fractal models for the roughness of rock fracture surfaces, have been postulated over the years for roughness characterization of rock fractures. Some of the most recent research in this direction is reported in Fardin et al. (2001) [527], Kwaśniewski and Wang (1997), Panagouli et al. (1997), Homand et al. (2001), Roko et al. (1997), Yang and Chen (1999), Belem et al. (2000), Yang et al. (2001a–c), Lanaro (2000), and Fu et al. (2001) [643–653]. Use of fractals to represent the roughness of rock fractures has become an important subject, as indicated in the above publications, but still remains a controversial topic in debate, as also in other fields (Whitehouse, 2001) [654].

Since the coupled hydro-mechanical effects become a more and more important aspect in rock mechanics studies, the constitutive models for aperture-flow relations and their coupling to mechanical displacements and damage of rock fractures become increasingly reported in literature. Although the dominant trend is still the idealized parallel plate model (Snow, 1965) [534] or the Cubic Law, in computer codes alternative models considering coupled stress-flow behaviour have been postulated over the years based on extensive experimental evidence, as reported by Gangi (1978), Walsh and Grosenbaugh (1979), Witherspoon et al. (1980), Walsh (1981), Tsang and Witherspoon (1981), Raven and Gale (1985), Brown and Scholz (1985), Brown (1987), Pyrak-Nolte and Cook (1988), and Cook (1988) [655–664]. More recent work is reported by Olsson (1992), Olsson and Brown (1993), Ng and Small (1997), Oron and Berkowitz (1998), Power and Durham (1997), Nicholl et al. (1999), Yeo et al. (1998), Indraratna et al. (1999), Pyrak-Nolte and Morris (2000), and Olsson and Barton (2001) [665–674].

Besides the fractures, there are other interfaces in rock engineering projects, such as interfaces between different materials (rock and soil, rock and buffer or backfilling material, rock and reinforcement elements, e.g. bolts, grouts, cables, etc.). Despite their obvious importance for the design and performance of rock engineering structures, development of special constitutive laws and models, and reporting of progress in this direction seems to be lacking. Examples of such models are reported in Fakharian and Evgin (2000) [675], and Cox and Hermmann (1998, 1999) [676,677] for steel–concrete bonds.

Although tremendous effort has been paid to develop constitutive models for rock fractures, the currently available models still cannot predict fracture behaviour with a reasonable level of confidence. The major difficulty is the lack of adequate understanding of the basic physics of the rock fractures, a unique and quantitative representation of joint roughness, damage evolution during a general deformation process, and its impact on the mechanical, hydraulic and thermal behaviour and properties of rock fractures. Other difficulties include the models for fractures at large scales, such as faults or fracture zones with large widths, time-scale dependence, and hydro-mechanical coupling effects. New subjects are the effects of chemical coupling and transport properties of fractures, as affected by understanding the flow path tortuosity, initial contact area and its evolution, and the fracture-rock interaction in terms of flow and transport during the deformation process of rock fractures. All the above add to the ever-increasing complexity in modelling the physico-chemical behaviour and properties of rock fractures, and introduces difficulties for formulating realistic constitutive models. On the other hand, it should also be noted that many models are developed for understanding the overall behaviour and help to solve practical problems. Therefore proper and prudent simplifications and idealization will always be needed to derive conceptualization models comprehensive enough for the problems at hand, while still retaining the necessary levels of scientific sophistication so that basic laws of physics and chemistry will not be violated.

4. Coupled thermo-hydro-mechanical models

The couplings between the processes of heat transfer, fluid flow and stress/deformation in fractured rocks has become an increasingly important subject in rock mechanics and engineering design since the early 1980s (Tsang, 1987, 1991) [678,679], mainly due to the modelling requirements for the design and performance assessment of underground radioactive waste repositories, and other engineering fields in which heat and fluids play important roles, such as gas/oil recovery, hot-dry-rock thermal energy extraction, contaminant transport analysis and environment impact evaluation in general. In fact, the coupling can be extended to include chemical and biosphere factors, but we concentrate here on the THM coupled models.

The term 'coupled processes' implies that one process affects the initiation and progress of another. Therefore, the rock mass response to natural or man-made perturbations, such as construction and operation of a nuclear waste repository, cannot be predicted with confidence by considering each process independently. The combination of the natural and engineering factors comprises also a coupled system (Hudson 1992; Jiao and Hudson, 1995) [680,681], the evolution of which involves a variety of mechanism pathways. For the THM model, it is certainly necessary to study the two-way interactions between the T, H and M components, as indicated in an outline form in Fig. 21.

Such mathematical models and associated computational methods are the only quantitative means for scientists and engineers to gain understanding of complex physical systems, often using multiple stochastic system realizations and parameter sensitivity analysis to account for the interactions among so many processes, properties and parameters, plus the uncertainty of parameter values. The complexity of the THM problem is increased by the presence of the rock fractures of various dimensions, whose physical behaviour under thermal, hydraulic and mechanical loadings is far from clearly understood, due mainly to the mostly unpredictable geometrical complexities of their surfaces.

The coupled THM process is mainly described by mechanics of porous media, which is applicable to fractured rocks. The first theory is Terzaghi's 1-D consolidation theory of soils (Terzaghi, 1923) [682], followed later by Biot's theory of isothermal consolidation of elastic porous media, a phenomenological approach of poroelasticity (Biot, 1941, 1956) [683,684], and the mixture theory by Morland (1972) [685], Bowen (1982) [686] and others. Non-isothermal consolidation of deformable porous media is the basis of modern coupled THM models using either averaging approach as proposed first by Hassanizadeh and Gray (1979a, b, 1980, 1990) [687–690] and Achanta et al. (1994) [691], or an extension to Biot's phenomenological approach with a thermal component (de Boer, 1998) [692]. The former is more suitable for understanding the microscopic behaviour of porous media and the latter is better suited for macroscopic description and computer modelling.

The subject attracted a very active research because of its wide reaching impacts in mechanics and engineering in geomaterials. Extensive research and publications have been generated. The fundamentals are systematically presented in many written or edited volumes such as in Whitaker (1977), Domenico and Schwartz (1990), Charlez (1991), Charlez and Keramsi (1995), Coussy (1995), Selvadurai (1996), Lewis and Schrefler (1988, 1998), Bai and Elsworth (2000) [693–701], and Sahimi (1995) [379] with focus on multiphase flow and

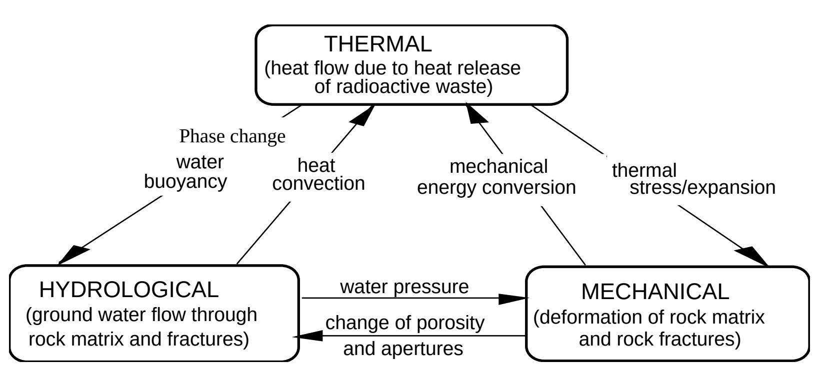

Fig. 21. Coupled thermo-hydro-mechanical processes in a fractured rock mass.

transport in porous media, targeting more for reservoir and environmental engineering applications. Tsang (1987) [678] and Stephansson et al. (1996) [702] are more focused on stress/deformation, fluid flow and heat transfer in fractured rocks, targeting especially for nuclear waste disposal applications.

THM coupling models have been developed according to two basic 'partial' coupling mechanisms which are well established within the principles of continuum mechanics: thermo-elasticity of solids (T–M) (interaction between the stress/strain and temperature fields through thermal stress and expansion) and poroelasticity theory (H–M) (interaction between the deformability and permeability fields of porous media). They are, in turn, based on the Hooke's law of elasticity, Darcy's law of flow in porous media, and Fourier's law of heat Conduction. The effects of the THM coupling are formulated as three inter-related PDEs expressing the conservation of mass, energy and momentum, for describing interactions among fluid flow, heat transfer and deformation processes.

The solution of the coupled sets of conservation equations can use either continuum or discrete approaches. For continuum approach, FEM, BEM, FDM and FVM are usually applied (Noorishad et al., 1992; Noorishad and Tsang, 1996; Rutqvist et al., 2001a, b; Börgsson et al., 2001; Nguyen et al., 2001; Pruess, 1991; Millard, 1996; Ohnishi and Kobayashi et al., 1996) [703–711]. A general framework of the equations and FEM formulation for porous media is given in Schrefler (2001) [712]. The continuum solution approach is based on the established equivalent properties of the fractured porous media. It is not computationally efficient when a large number of fractures are explicitly represented and derivation of whose equivalent properties, especially their scale-dependency, often needs discrete numerical methods.

The numerical methods for THM processes using discrete approach have not reached the same degree of maturity compared with their continuum counterparts, mainly because the fluid flow is most often limited only in fractures and matrix flow, therefore also the fracture–matrix interaction, is not considered. The most representative example of the discrete numerical method for coupled THM processes in fractured rocks is the UDEC/3DEC DEM code group. Heat convection can be considered (Abdaliah et al., 1995) [713], but partial saturation and fluid phase change have not been incorporated yet, because no fluid is assumed in the matrix.

Comprehensive studies, using both continuum and discrete approaches, have been conducted in the international DECOVALEX[§3] projects for coupled THM processes in fractured rocks and buffer materials for underground radioactive waste disposal since 1992. Results have been published in a series of reports (Jing et al., 1996, 1999) [714,715], an edited book (Stephansson et al., 1996) [702] and two special issues of the International Journal of Rock Mechanics and Mining Sciences (Stephansson, 1999; Stephansson et al., 2001) [716,717]. These contribute greatly to the understanding of the coupled processes and the mathematical models.

Coupled processes have also been studied in other application areas, such as:

- Reservoir simulations: Gutierrez and Makurat (1997) [269], Zhao et al. (1999), Yang (2000), Sasaki and Morikawa (1996) [718–720];
- Partially saturated porous materials: Gawin and Schrefler (1996) [721];
- Advanced numerical solution techniques for coupled THM models: Wang and Schrefler (1998), Cervera et al. (1996); Thomas et al. (1999) [722–724];
- Soil mechanics: Thomas and Missoum (1999) [725];
- Simulation of expansive clays: Thomas and Cleall (1999) [726];
- Flow and mechanics of fractures: Selvadurai and Nguyen (1999) [727];
- Nuclear waste repositories: Selvadurai and Nguyen (1996), Hudson et al. (2001) [728,729];
- Non-Darcy flow in coupled THM processes: Nithiarasu et al. (2000) [730];
- Double-porosity model of porous media: Masters et al. (2000) [551];
- Parallel formulations of coupled hydro-mechanical and thermo-mechanical models for porous media: Zimmerman (2000) [731];
- Tunnelling in cold regions: Lai et al. (1998) [732].

In the face of ever increasing complexity in the modelling of large numbers of concurrent and consecutive operating mechanisms and mechanism pathways in the coupled models and with many geometrical and parameter uncertainties, we can foresee three directions which future coupled modelling may take:

- continue with the current direction, refining the algorithms and using sensitivity analyses to establish the significance of geometrical and parameter uncertainties;
- continue with the current direction, but extend the models to include further physical and chemical/biochemical mechanisms, concentrating on equivalent geometries and 'first order' mechanism effects;
- change the modelling paradigm to 'non-1:1 mapping' (cf. Fig. 1 and the discussion in Section 3.7).
- develop solution techniques so that large-scale equation systems can be solved more efficiently for coupled equation systems.

[3] DECOVALEX (acronym for DEvelopment of COupled models and their VALidation against EXperiments.

These anticipated future developments are all based on extensions of current activities, and will be facilitated by continuing advances in computing capabilities.

The third and fourth developments are anticipated because the amount of information used in modelling (measured in bits) has greatly increased from the 1940s to the present day, from just a few bits in the first calculations to perhaps terrabits (10^{12}) and exabits (10^{15}) nowadays when all the information in a computerized time-marching operation is considered. Such exponentially increasing curves always collapse at some stage, and this difficulty will only be overcome through more efficient form of simplification, probably by a switch of the modelling paradigm from '1:1 mapping' to 'non-1:1 mapping' of the geometry and mechanisms, and more efficient calculation techniques, such as parallel processing.

5. Inverse solution methods and applications

A large and very important class of numerical methods in rock mechanics and civil engineering practice is the inverse solution techniques. The essence of the inverse solution approach is to identify unknown system properties or perturbation parameters, through direct application of numerical methods or closed-form solutions to derive unknown material properties, system geometry, and boundary or initial conditions, based on a limited number of measured values of some key variables, using either least-square or mathematical programming techniques of error minimization. The technique has been long applied in rock characterization practices, such as stress measurements using over-coring or hydraulic fracturing (by deriving stress data from measured displacements) and underground structure identification (by deriving geometry of structures using ground penetration radar images). Because the aim of the solution is the system parameters (which should be known beforehand in ordinary forward simulations), thus the name 'inverse solution'. In the case of rock engineering, the most widely applied inverse solution technique is back analysis using measured displacements from convergence extensometer or fluid pressure data to derive mechanical and hydraulic properties of rock, such as deformability and permeability in general civil/hydrogeological engineering. The technique was initiated by Sakurai (1981) for displacement back-analysis and has been extensively applied in rock engineering.

The numerical models for forward analysis are generally a closed system with all necessary assumptions about constitutive models and parameter values and initial/boundary conditions. The input data and output variable values have a one-to-one response and the solution is unique, although the output results may or may not agree with the measured variable values, depending largely on the correctness of the model conceptualization concerning material behaviour and system geometry, especially effects of fractures. Such uniqueness in solution, however, is not guaranteed in inverse solutions, even when the constitutive models are assumed, since an acceptable agreement between the numerical results and measured data may be obtained with different forms of constitutive models and multiple fracture system realizations.

Fig. 22 illustrates the concepts of the forward and back analysis applied in general geotechnical engineering (Sakurai, 1997) [733]. Back analysis can be performed with assumed constitutive laws, usually called parameter identification. However, forms of constitutive laws themselves may also be the objects of back analysis, at least in theory, as pointed out by Sakurai and Akuragawa (1995) [734], which may not be easily achieved in practice.

A distinct advantage of back analysis is the fact that the measured values in the field represent the behaviour of large volumes of rock masses containing effects from largely unknown fractures: the scale effects of the constitutive parameters are automatically included in the identified parameter values without burden of prove for the existence of the REV for fractured rocks. It also points to a promising method for validation of constitutive models and properties using back analysis with field measurements. On the other hand, the same objective can also be reached by using successive forward solutions with material property perturbations through a global optimization process or mathematical programming.

Since groundwater flow has become an important aspect in numerical modelling work in rock mechanics and rock engineering, it is necessary to present briefly the advances and status of the subject. More comprehensive summary and in-depth analysis of the subject is beyond the scope of this review.

In this Section, brief summaries of the principles and applications of two main inverse solution techniques, the displacement-based back analysis for rock engineering and pressure-based inverse solution for groundwater flow analysis, are presented to demonstrate the history and trends of development of this particular technique.

5.1. Displacement-based back analysis for rock engineering

Since displacements along extensometers with multiple anchors and the convergence of tunnel walls are the most directly measurable quantities in situ, and are one of the primary variables in many numerical methods, they have been extensively used to derive rock properties over the years. The majority of applications concern

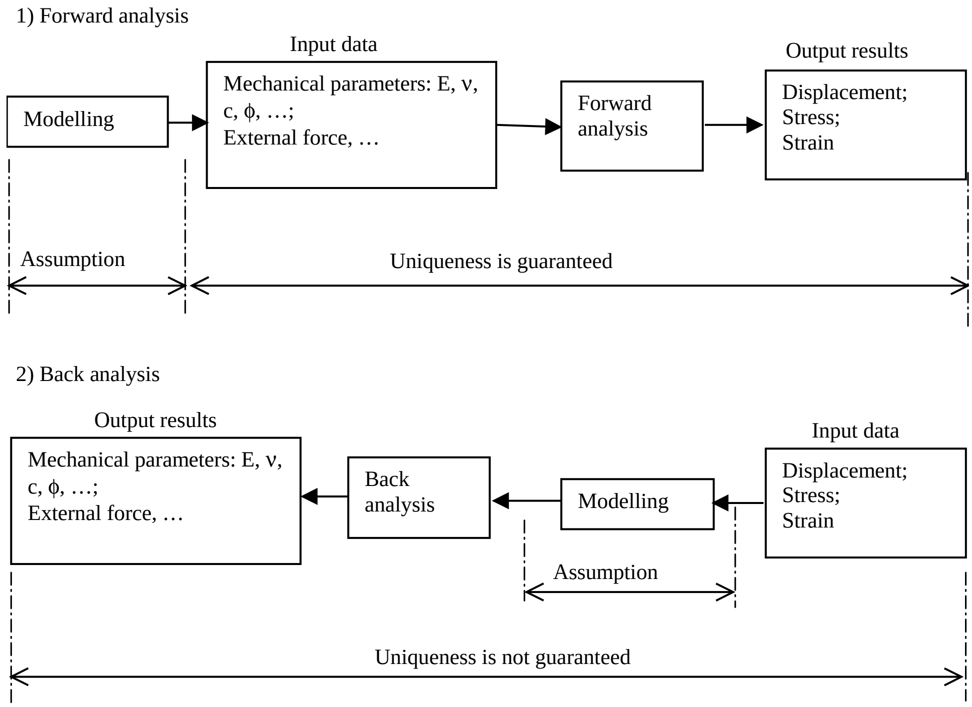

Fig. 22. Comparison between the procedures of forward and back analyses (Sakurai, 1997) [739].

identification of constitutive properties and parameters of rocks, using displacement measurements from tunnels or slopes. Below are some typical and recent examples of such applications:

- Underground excavations: Gens et al. (1996), Ledesma et al. (1996a, b), Mello Franco et al. (1997), Hojo et al. (1997), Sakurai (1997), Singh et al. (1997), Pelizza et al. (2000), Yang et al. (2000), Krajewski et al. (2001), Chi et al. (2001), Hatzor and Benary (1998), Yang et al. (2001) [734–744,300,517];
- Slopes and foundations: Ai-Homoud et al. (1997), Okui et al. (1997), Sonmez et al. (1998), Feng et al. (2000), Deng and Lee (2001) [745–747,450,454];
- Initial stress field: Yang et al. (1996) [748], Rossmanith and Uenishi (1997) [588];
- Stress measurements: Obara et al. (2000) [749];
- Time-dependent rock behaviour: Ohkami and Swoboda (1999) [333], Guo (2000) [750];
- Consolidation: Kim and Lee (1997) [751].

5.2. Pressure-based inverse solution for groundwater flow and reservoir analysis

Inverse solutions have long been used in hydrogeology, reservoir engineering (oil, gas and Hot-Dry-Rock geothermal reservoirs) and geotechnical engineering practice of environmental impacts, as a critical technique for estimating hydraulic properties of large-scale geological formations. Similarly, the fluid pressure values measured from boreholes/wells are used to identify the hydraulic properties of rock formations, such as permeability, porosity, storativity, etc. by assuming hydraulic constitutive laws of porous media based on Darcy's law or other non-Newtonian fluid models. Complexity is increased when the thermal process is involved due to the additional parameters of thermal diffusivity involving phase change of multiple-phased flow of fluids, with various states of saturation. However, the mathematical principles and the issue of solution uniqueness are the same as that of the displacement-based back analysis for rock mechanics. Inverse solutions against laboratory measurement results are also often used for parameter identification of rock/soil matrices, and fractures, and the upscaling of hydraulic parameters has been a very active issue in hydrogeology with a vast number of publications.

A comprehensive review of the subject is given in de Marsily et al. (1999) [752] for the history of inverse solution development and methods for the past 40 years, especially the application of the stochastic approach using geostatistics. Due to the space limit, only some most recent developments in this subject are quoted below to illustrate the advances of the development:

- Capillary pressure-saturation and permeability function of two-phase fluid in soil: Chen et al. (1999) [753];

- Water capacity of porous media: Fatullayev and Can (2000) [754];
- Unsaturated properties: Finsterle and Faybishenko (1999) [755];
- Transmissivity, hydraulic head and velocity fields: Hanna and Yeh (1998) [756];
- Hydraulic function estimation using evapotranspiration fluxes: Jhorar et al. (2002) [757];
- Use of BEM for inverse solution: Katsifarakis et al. (1999) [758];
- Integral formulation: Vasco and Karasaki (2001) [759];
- Hydraulic conductivity of rocks using pump test results: Lesnic et al. (1997) [760];
- Least-square penalty technique for steady-state aquifer models: Li and Yang (2000) [761];
- Maximum likelihood estimation method: Mayer and Huang (1999) [762];
- Inversion using transient outflow methods: Nützmann et al. (1998) [763];
- Use of geostatistics for transmissivity estimation: Russo (1997), Roth et al. (1998), Wen et al. (1999, 2002) [764–767];
- Successive forward perturbation method: Wang and Zheng (1998) [768].

The inverse solution is closely related to the constitutive models, especially to identification of model parameters and upscaling, as demonstrated in Lunati et al. (2001) [544]. It is also widely applied to other branches of mechanics and material sciences, such as heat transfer (Chapko, 1999) [769], stress–strain inversion without assuming constitutive models for solid mechanics (Hori and Kameda, 2001) [770] and non-iterative least-square inversion algorithm (Shaw, 2001) [771]. With the complex and largely unknown fracture systems and their significant impact on the physical behaviour of fractured rocks, back analysis and inverse solutions appear to play more and more important roles in modelling, design and performance assessments of rock engineering problems since the directly measured data are not only the most reliable, but also the only available responses one can obtain from rocks in situ as a result of man-made perturbations.

6. Advances and outstanding issues

This Review began by describing the special features of rock masses, the difficulties of characterizing their DIANE nature, and presenting an overview diagram of rock mechanics modelling approaches. The various numerical modelling techniques were then described in some detail with comments on their applicability, followed by a Section on the way in which coupled models are being developed. It has also been intimated that, after the past 50 years' development, we still mainly use empirical failure criteria, empirical design procedures, and an observational approach to rock engineering. There have been many significant advances in numerical methods, including developments that allow the DIANE features of a rock mass to be captured in the models. However, there are still outstanding issues relating to the numerical models themselves and to the utility of the models for supporting rock engineering practice.

6.1. Advances in numerical modelling in rock mechanics

To adequately capture the rock reality and the perturbations introduced by engineering, a numerical model for the design and performance analysis of rock engineering projects should have the capability to represent the system geometry (including the fracture system geometry), boundary and initial conditions, any natural and induced loading/perturbation histories, complex material models (constitutive laws) for both the rock matrix and fractures, including scale-time effects, uncoupled or coupled physical processes, complex construction sequences, interactions between all system components and interfaces—and all in both 2-D and 3-D spaces.

Such 'all-encompassing' numerical models do not exist today, mainly because of our limited knowledge about the physical behaviour of rock fractures and fractured rock masses, our limited means to represent the geometry and evolution of complex rock fracture systems, and our limited computational capacity for large or very large-scale problems. For the more complex rock engineering problems, numerical modelling is still largely a tool for conceptual understanding and generic studies. However, for 'simpler' rock engineering projects, such as tunnel design and slope stability analysis, numerical modelling has become a valuable design tool.

Indeed, our advances in the field of computational methods in rock mechanics since the dawn of the computer era in civil engineering fields over the last three decades is impressive. These advances can be briefly summarized as follows.

- Parallel development of continuum-based and discrete numerical models, such as FEM, BEM, and DEM, for stress/deformation analyses. The discrete models are a special development, driven by the need of rock and soil mechanics models in the early 70s (when the early FEM and BEM codes started to be applied to rock and soil mechanics problems, based almost entirely on linear elasticity). The result was a drive for much greater in-depth understanding of the mechanical behaviour of fractured rock masses and granular soils due to the DEM's ability to provide a more realistic representation of the rock and soil fabrics and significant improvement of numerical modelling at moderately large scales. Special credit should be given to P.A. Cundall and G. Shi, and their co-workers, for the explicit and implicit DEM, DDA and Key Block theory approaches.

- Significant improvement of traditional FEM, DEM and FEM techniques for rock mechanics problems, especially regarding fracture simulations with developments such as meshless models (cf. Section 3.2), enriched FEM (cf. 3.2), GBEM (cf. 3.3) and FVM (cf. 3.1). These advancements not only enriched the numerical methods themselves, but also significantly improved our computing capability and overcame one of the most significant difficulties—the simulation of fracture evolution. Special credit should be given to T. Belytschko and his co-workers for the development of the meshless technique that has become an important field of research and application for fractured rocks.
- Significant advances in the mapping and representation of the fracture systems in fractured rocks by stochastic DFN models. This advance is a specially useful development for fractured hard rocks and has been developed and applied to a large variety of problems where fluid flow is the main concern. Although severe limitations still exist (see later in this section), the DFN has been developed from simple generic study techniques into a practical tool for engineering scale applications in a relatively short time, less than 20 years. Special credit should be given to J.C.S. Long, P.A. Robinson, W.S. Dershowitz, A.W. Herbert and others for initiation and continuous development of the DFN codes.
- Significant improvements in the formulation of complex constitutive models for rocks, soils and rock fractures, using different mathematical platforms, such as plasticity theory, damage mechanics, contact mechanics and fracture mechanics. The second law of thermodynamics is being applied more and more in the model formulations, either implicitly or explicitly, and empirical relations obtained from laboratory tests are used more correctly. The formulation of constitutive models for numerical methods has been largely driven by the need to characterize rocks and rock fractures including stress-dependency, scale-dependency, time-dependency, roughness characterization, damage and degradation, contacts and their evolution, strain-localization, bifurcation, etc. Any one of these problems is a serious challenge: the combination of all such problems, especially concentrated when characterizing fractured rocks, makes rock mechanics one of the most challenging fields of applied mechanics.
- Development of alternative numerical models, often as companions to the main-stream methods, such as the BCM accompanying the BEM, lattice model accompanying the DEM, percolation theory accompanying the DFN, etc. Although the roots of these alternative formulations can be traced to the early 40s and 50s, their applications to rock and soil mechanics fields are relatively new, and they have played a significant role in computational mechanics for civil engineering.
- Development of mathematical models and computer codes for coupled THM-processes of fractured geological media, mainly fractured rocks and buffer materials, and their verification against small, medium and large-scale in situ experiments and generic benchmark test problems, since the late 80s. This development greatly broadens the scope of the traditional rock/soil mechanics disciplines, and the fields have been developed more systematically under the governance of fundamental physical laws, such as mass, momentum and energy conservation, and concerning more general physical processes such as fluid flow, heat transfer and stress/deformation, rather than empirical approaches concerning only strength/failure phenomena. The discipline of rock mechanics is now established on a much improved scientific foundation compared with 20 years ago. Special credit should be given to C.-F. Tsang, J. Noorishad, and the international DECOVALEX project.
- Development of successful commercial software packages which can be used by both the research community and practising engineers, such as the UDEC/3DEC/PFC DEM code group, FracMan/MAFIC DFN codes, FLAC-2D and -3D FDM code groups, etc. The popularity of these software packages is largely due to their user-friendliness and PC-oriented operating environment, which enables the practising engineers, students and researchers to run the models quickly without going through the special training required and the inconvenience when large, main-frame computers are needed. This user-friendliness is one of the important factors for the wide propagation of numerical modelling in civil engineering fields today.

Despite all the advances, our computer methods and codes can be inadequate when facing the challenge of practical problems, especially when adequate representation of rock fracture systems and fracture behaviour are the pre-conditions of successful modelling. The fundamental reason for this is that there are still many factors beyond the comprehension and hence control of researchers, code developers and practising engineers alike. The issues of special difficulty and importance are presented below.

6.2. Issues of special importance and difficulty in numerical modelling for rock mechanics

Although the basic principles and engineering practice of rock mechanics and engineering are well known today, and a large number of numerical codes has been developed over the years for design and simulation purposes, a number of important scientific and technical issues of difficulty exist today in either fundamental understanding or numerical implementation.

(1) *Understanding and mathematical representation of large rock fractures*. It is common practice that rock fracture properties are obtained from laboratory tests on samples of smaller fractures of limited size, say 100–400 mm. This size may or may not be large enough to reach the stationarity threshold of even the smaller fractures, depending on the roughness characteristics of the sample surfaces (see, for example, Fardin et al., 2001) [527]. There is an acute lack of understanding of the hydro-mechanical behaviour of large fractures (such as faults or fracture zones) with a large width (say, 10 mm–50 m). This type of rock fracture has been associated with problems in many engineering projects, and they are the most important geologic feature for the design and safety of radioactive waste repositories.

The difficulty is the fact that laboratory scale tests cannot be applied to establish the characteristics of large-scale fractures, and in situ experiments are difficult to control in terms of loading and boundary conditions, and are expensive and difficult to 'decode'. The lack of progress in this regard leads also to the lack of proper constitutive models for the large fracture zones, which, in turn, reduces our current capacity in numerical modelling for large-scale problems. Possible remedies may be the use of back-analysis using large-scale in situ instrumentation information for underground works or GPS monitoring data for surface works—but the derived properties are still subject to the validity of the assumed constitutive models.

(2) *Quantification of fracture shape, size and connectivity for DFN and DEM models*. Measuring the shape, size and connectivity of fractures is an extreme technical difficulty, but the parameters are critical in characterizing the fracture system geometry and play important roles in the permeability of the fractured rock mass. The difficulty lies in two critical aspects: the size (obtained from trace length information) and fracture behaviour (obtained largely from laboratory tests). Thus, the quality of DFN models depends directly on the quality of the field mapping, which is affected by a large number of factors (such as available exposure areas of the mapping sites, limits of window/scanline mapping regarding trace length biases, etc.), and the unknown effect of cut-off limits of the mapping on the validity of the DFN models. The effects of fracture intersections for both flow and stress/deformation is much less understood, compared to the mechanics and flow of single fractures. Because there is unlikely to be a rapid technical breakthrough in mapping technology, the possible remedy is to reduce (or at least understand) the uncertainties by using Monte Carlo processes with DFN models and systematic analyses of effects of the fracture geometry assumptions. This will provide a method for evaluating uncertainty and variability ranges.

(3) *Representation of rock mass properties and behaviour as an equivalent continuum*. Since most of the measured rock properties are obtained through small-scale laboratory tests, the measured values are at best valid only at these small scales, representing only intact rock matrix or single small fractures. For large-scale problems, rock masses are often assumed to be equivalent continua, and the equivalent properties therefore need to be evaluated mathematically; this is the termed upscaling and homogenization of the rock mass properties which is necessary for numerical solution of the problems—but is subject to the validity of some crucial assumptions:
   (a) existence of an REV which may or may not exist in fractured rocks, depending on variations in the fracture density and geometry, model cut-off limits and computing power limitations;
   (b) assumption of constitutive behaviour of the equivalent continuum, which may or may not be valid;
   (c) assumption of the constitutive representations for rock matrix and fractures, which may or may not be sufficiently comprehensive;
   (d) effects of fracture intersections whose hydro-mechanical effect is itself a difficult and unsolved issue, especially in 3-D and for fluid flow problems; and
   (e) the problem is site-specific and general solution techniques may or may not exist.

   Large-scale block tests or in situ borehole or pressure-tunnel tests have similar drawbacks relating to the evaluation of fracture effects because the precise fracture geometry is also unknown in these tests. Therefore rock mass properties remain an issue of particular difficulty. Numerical models were developed using the discrete models for deriving equivalent hydro-mechanical properties, mostly for 2-D cases. Three-dimensional analysis is rarely reported mainly because of computational difficulties in realistic fracture geometry representations.

(4) *Numerical representation of engineering processes, such as excavation sequence, grouting and reinforcement.* It is not sufficient to consider just the initial and final states for rock engineering: it may be necessary, for example, to consider the complete stress path during the stages of an excavation. This is important for mining engineering, where there are often complex geometry changes, but can be equally important for civil engineering projects where there is a set of excavation sequences and/or complex excavation geometries. Also, if grouting and reinforcement measures are to be included for analysis, e.g. evaluation of long-term stability contributions from reinforcement measures, these measures need to be included in the capabilities of the computer codes, with properly developed constitutive models for not only the supporting elements and materials, but also contacts between the rock and reinforcement. This is a difficult area in current rock mechanics and engineering and requires considerable increase of knowledge and computing power, especially for 3-D models. Empirical approaches have been especially successful in the face of such difficulties. As Barton has said (2001), "We need to damp the model down in order to make it work." [772].

(5) *Interface behaviour*. There are different types of interfaces between components of a rock engineering project. Most notable in conventional rock engineering are the interfaces between the rock reinforcement and support components and the rock mass. For a radioactive waste repository, the most notable interfaces are threefold: canister-bentonite, bentonite-rock and backfill-rock. An adequate understanding of the THM behaviour of these interfaces is needed for evaluation of their functions for repository design and performance assessment. However, there is no experimental basis today for this understanding and it is an acute need for research, especially regarding the bentonite-rock and backfill-rock interfaces.

(6) *Large-scale computational capacities.* Most of today's numerical codes for rock mechanics and rock engineering are suitable only for small scale generic studies, mostly in 2-D. A few large-scale commercial codes, such as ABAQUS, DIANA, etc. are standard structural analysis codes and lack important capabilities for rock mechanics and engineering problems, such as fracture system treatment, constitutive models for rock fractures, construction/support sequence simulation, and THM coupling. Enhanced computing power is required for large-scale applications in the near future and so development using parallel solution techniques is needed.

(7) *Scale and time effects*. These two effects are the most often-mentioned difficulties in rock engineering modelling because they are difficult to investigate by either experimental or mathematical means. The scale effect is caused by scale-dependency of physical properties and the geometry of fractures; and the time effect is caused mainly by material creep over long periods of time. Another type of time effect is the dynamic behaviour of a rock excavation, such as during earthquakes. The difficulty lies not only in understanding and mathematical models for dynamic systems, but also in obtaining dynamic values of all material properties, which are usually measured in static conditions.

(8) *Systematic evaluation of geological and engineering uncertainties*. Although numerical modelling has incorporated uncertainties, especially geological uncertainties such as uncertain fracture system geometry and fracture behaviour, the systematic mathematical evaluation of the effects of such uncertainties has not been attempted in the field of rock mechanics. Perhaps some uncertainties, such as fracture size and shape, can never be truly evaluated. On the other hand, mathematical tools exist today for partial quantitative evaluation of uncertainties, such as uncertainty intervals, probability bounds, fuzzy sets, neural networks and hybrid arithmetic and fuzzy geostatistics (Banks, 2001) [773]. They may become useful complements to the traditional 'engineering equivalence' approaches as commonly applied today in rock engineering for reducing property variations and raising model confidence.

The above issues cannot cover all points of difficulty in numerical modelling in rock mechanics. Other issues of significance are the need for more laboratory and in situ experiments for verification of numerical methods, codes and models with well-controlled testing conditions and large enough sample sizes, and the need for more efforts in the combined applications of different modelling approaches, especially the 1:1 and not 1:1 modelling approaches, such as using rock mass classification with more numerically based methods.

7. Conclusions

The conclusions are presented in two parts: firstly, specific conclusions relating to the main numerical modelling methods; and then overview comments relating to the general subject of numerical modelling in rock mechanics.

7.1. Main numerical modelling methods

7.1.1. Finite Element Method

The FEM will remain a mainstream numerical tool in engineering computations for the visible future, simply because of its maturity and advantages in handling material inhomogeneity and non-linearity, and the availability of many well-verified commercial codes for large- or small-scale problems. Moreover, the meshless, manifold or generalized FEM approaches may become important subjects for further research and development for problems of fractured rocks, mainly because of their flexibility for meshing and capability of fracture evolution simulations without re-meshing. This fracture simulation development provides a challenge to the advantage, enjoyed so far by the BEM approach for fracturing process simulation, but with the added advantage of more natural treatment of material non-linearity.

7.1.2. Boundary Element Method

The BEM method remains today the best tool for simulating fracturing processes in rock and other solids. The Galerkin BEM approach provides also a promising platform for overcoming corner problems and coupling with FEM because of its symmetric stiffness matrix. The BEM's advantage of smaller computer memory and block-like matrix structure when the multi-region technique is used makes it more suitable for solving large-scale problems with reduced degrees of freedom, compared with the FEM and FDM. Algorithms for coupled TM and HM processes have been developed without fracturing processes being considered, but modelling fully coupled THM processes using BEM technique has yet to be developed.

7.1.3. Finite Difference Method/Finite Volume Method

The FDM, especially FVM, remains a powerful numerical tool, not only because of its conceptual simplicity, but also its flexibility in handling material non-linearity. Its coupling with contact mechanics for deformable block systems produced the DEM approach. It is especially useful for solving fluid flow equations, and is therefore a useful tool for coupled THM problems of large scale—due to its smaller matrix size using iterative solution techniques.

7.1.4. Discrete element method (DEM)

The DEM approach, either explicit or implicit, has become a powerful numerical modelling tool simply because of its flexibility in handling a relatively large number of fractures, for either purely mechanical problems or for coupled THM processes. The main shortcoming for the latter is the lack of fluid flow in the rock matrix, so that the matrix–fracture flow interaction cannot be adequately treated. The reason is the exceptional computational effort, both computer memory and running time, required for even a moderately large number of blocks. Coupling with meshless BEM, and using parallel processing techniques on main-frame computers may be useful alternatives to extend the DEM's capacity. However, the main difficulty for the DEM is the uncertainty about the fracture system geometry, and the effect of this uncertainty is difficult to quantify. In addition, a DEM model requires 3-D simulation: 2-D models can only be used for generic studies, or when the fracture system geometry and relative orientations of the engineering structures permit 2-D simplifications without causing unquantifiable errors.

7.1.5. Discrete Fracture Network model

Like the DEM, the DFN model was developed from a need to represent more realistic fracture system geometries in 3-D (with 2-D DFN models having limited practical applicability). And also like DEM, the DFN models suffer the same shortcomings relating to uncertain fracture system geometry. However, the DFN approach is a valuable tool for generic studies for quantitatively evaluating the impact of fracture system variations on the model output. Large-scale DFN calculations are easier to run because the number of degrees of freedom of DFN models is much less compared with FEM—because the discretization for 3-D problems is essentially at 2-D (if the FEM mesh is used for fracture discretization) and 1-D (when pipe and lattice network models are used). The problem of fracture intersections, such as mixing of flow and transmissivity change due to stress and displacement discontinuities at intersections, has yet to be properly solved, especially when the fracture deformation is included. These disadvantages of lack of matrix flow and stress influence may be difficult to overcome in the near future, especially for near-field problems. Understanding and quantifying the geological and physical uncertainties remains the main task for DFN development.

7.2. General comments regarding numerical models

1. The 'model' and the 'computer' are now integral components in studies for rock mechanics and rock engineering. Indeed, numerical methods and computing techniques have become daily tools for formulating conceptual models and mathematical theories integrating diverse information about geology, physics, construction technique, economy and their interactions. This achievement has greatly enhanced the development of modern rock mechanics from the traditional 'empirical' art of rock strength estimation and support design to the

rationalism of modern mechanics, governed by and established on the three basic principles of physics: mass, momentum and energy conservation.

2. The characteristics of the rock mass that distinguish it from other engineering materials are the rock mass fracturing and inhomogeneity that occurs at several scales. Rock masses are disordered fractured media in which multiple physical and chemical processes co-exist and interact, and the parametric values of the main feature, the fracture system, are and will remain largely unknown. Consequently, the major thrust of research and development in rock mechanics is characterization and representation of the rock fractures, both as individual entities and as a collective system. Furthermore, the lack of information about the rock fractures means that working with uncertainty and variability becomes a way of life in rock mechanics and rock engineering, which for numerical modelling demands clarification of sources, significances, propagation paths of uncertainties and their mathematical treatment.
3. It is in regard to the rock fracturing that numerical models play their most useful role: they can provide information on or insights into the behaviour of rock masses (that cannot be obtained by experiments or observation) by systematic stochastic, sensitivity and scoping analyses with chosen sophistication in processes, properties, parameters and engineering perturbations. The main advantage of the numerical models is their repeatability with clear demonstration of causes and effects.
4. Full validation of numerical models and computer codes by experiments in rock mechanics is not possible, and can at best be only partial, due to the necessary assumptions in the mathematical models and hidden nature of the fractures. All verifications are relative by nature, but our confidence in the numerical models can be raised when they are successfully calibrated against well-controlled laboratory and in situ experiments, and when their output for analysing practical problems follows both the basic laws of physics and engineering experience. This combined 'scientific' and 'engineering' support for judgement is essential for applying numerical methods to rock mechanics and rock engineering.
5. The most important step in numerical modelling is not running the calculations, but the earlier "conceptualization" of the problem regarding the dominant processes, properties, parameters and perturbations, and their mathematical presentations. The associated modelling component of addressing the uncertainties and estimating their relations to the results is similarly important. The operator should not 'dive in' and just use specific approaches, codes and numerical models, but first consider the specific codes and models to evaluate the harmony between the nature of the problem and the nature of the codes, plus studying the main uncertainties and their potential effects on the results. This practice is necessary because today many different approaches and computer codes are available and the operator must have a clear idea to harmonize the program to the problem. This idea is strongly related to technical auditing and quality control issues which will become increasingly applied in the future as the design of large rock engineering projects becomes more subject to independent audit (Hudson, 2001) [1,2].
6. Although clearly defined mathematical approaches may exist to describe and analyse uncertainties and error propagation, their application in mathematical and computer models of rock engineering is still difficult—simply because we do not have a reference basis for judgements, except for empirical judgement. Very often, mistakes are identified directly through structural failure or accidents, but conceptual failures and modelling mistakes can be hidden under the thick blanket of the 'operational success' of the rock engineering structure. Moreover, model reliability and credibility are always relative, subjective and case-dependent. This lack of a rigorous treatment of uncertainty in rock engineering may well be a major reason why many practising engineers and researchers remain uncertain about the usefulness of mathematical models and computer methods.
7. There are no clear-cut advantages or shortcomings when considering continuum or discrete models for simulating fractured rocks. Continuum models often include a limited number of explicitly represented fractures of usually larger scales, and the blocks in the discrete models are treated as continuum bodies with standard discretization meshes. The main difference is
   - whether contacts between the blocks (particles) remain unchanged (continuum approach) or need to be continuously updated using contact mechanics principles (discrete approach), and
   - whether the fractures are permitted to have large-scale displacement/movement, including rotation and complete detachment (discrete approach).

   Since the main effects of the fractures are mostly concentrated near excavated surfaces, the discrete approach is more appropriate for near-field representations and the equivalent continuum approach is more efficient for far-field regions. Hybrid models are therefore the natural product of combinations of the two.

8. Experiences with the comparison between the discrete and continuum approaches for coupled T–H–M processes for fractured rocks show that our present numerical ability can predict the heat transfer processes in fractured rocks, either with or without convection by fluid flow with high confidence, as demonstrated by a large number of benchmark tests, small and medium scale laboratory and large-scale in situ heating experiments. Our present numerical capability can also provide accepted predictions for stress and displacement of fractured rocks in terms of magnitudes with reasonable confidence. However, current numerical methods cannot predict fluid flow in fractured rocks with reasonable confidence, especially for the near-field, due mainly to the dominance of the, unknown, fracture system geometry, even when the DFN models are used (Jing et al., 1996) [714].
9. Finally in summary, success in numerical modelling for rock mechanics and rock engineering depends almost entirely on the quality of the characterization of the fracture system geometry, physical behaviour of the individual fractures and the interaction between intersecting fractures. The engineer needs a predictive capability for design, and that predictive capability can only be achieved if the rock reality has indeed been captured in the model—and the rock fractures dominate. Today's numerical modelling capability can almost handle very large-scale and complex equations systems, but the quantitative representation of the physics of fractured rocks remains generally unsatisfactory, although much progress has been made in this direction.

The key issues of importance for numerical modelling are:

- the development of advanced rock mass characterization techniques and modelling methods, and
- more scientific treatment of the sources and effects of uncertainties in geology, material behaviour, natural or man-made disturbances, and other economic and social constraints.

The fact that these uncertainties will remain and will never be truly removed is part of the character of the rock mechanics and rock engineering subjects. Researchers and practitioners should understand the uncertainties, scientifically present them, and assess them—so that the design and performance assessment of engineering works in fractured rocks can be performed with adequate management of the risks yet without being over-conservative.

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References

[1] Harrison JP, Hudson JA. Engineering rock mechanics. Part 2: illustrative workable examples. In: Särkkä P, Eloranta P, editors. Oxford: Pergamon, 2000.
[2] Hudson JA. Rock engineering case histories: key factors, mechanisms and problems. In: Särkkä, Eloranta, editors. Rock Mechanics—a challenge for society. Proceedings of the ISRM Regional Symposium EUROC2001, Espoo, Finland, 4–7 June 2001. Rotterdam: Balkema, 2001. p. 13–20.
[3] Lorig LJ, Brady BGH. A hybrid computational scheme for excavation and support design in jointed rock media. In: Brown ET, Hudson JA, editors. Proceedings of the Symposium Design and Performance of Underground Excavations. Cambridge: British Geotechnical Society, 1984. p. 105–12.
[4] Da Cunha AP. Scale effects in rock masses. Rotterdam: Balkema, 1990.
[5] Da Cunha AP. Scale effects in rock masses. Rotterdam: Balkema, 1993.
[6] Amadei B. Orally presented in the closing talk at Pacific Rocks 2000, Fourth NARMS Symposium, Seattle, 2000.
[7] Eberhardt E. Numerical modelling of three-dimensional stress rotation ahead of an advancing tunnel face. Int J Rock Mech Min Sci 2001;38:499–518.
[8] Hazzard JF, Young RP. Simulating acoustic emissions in bonded-particle models of rock. Int J Rock Mech Min Sci 2000;37(5):867–72.
[9] Wheel MA. A geometrically versatile finite volume formulation for plane elastostatic stress analysis. J Strain Anal 1996;31(2): 111–6.
[10] Perrone N, Kao R. A general finite difference method for arbitrary meshes. Comput Struct 1975;5:45–58.
[11] Brighi B, Chipot M, Gut E. Finite differences on triangular grids. Numer Methods Partial Differential Equations 1998; 14:567–79.
[12] Selim V. A node centred finite volume approach: bridge between finite differences and finite elements. Comput Methods Appl Mech Eng 1993;102:107–38.
[13] Fallah NA, Bailey C, Cross M, Taylor GA. Comparison of finite element and finite volume methods application in geometrically nonlinear stress analysis. Appl Math Modelling 2000;24:439–55.
[14] Bailey C, Cross M. A finite volume procedure to solve elastic solid mechanics problems in three-dimensions on an unstructured mesh. Int J Numer Methods Eng 1995;38: 1757–76.
[15] Fryer YD, Bailey C, Cross M, Lai CH. A control volume procedure for solving the elastic stress–strain equations on an unstructured mesh. Appl Math Modelling 1991;15:639–45.

[16] Wilkins ML. Calculation of elasto-plastic flow. Lawrence Radiation Laboratory, University of California, Research Report UCRL-7322, Review I, 1963.
[17] Taylor GA, Bailey C, Cross M. Solution of the elastic/visco-plastic constitutive equations: a finite volume approach. Appl Math Modelling 1995;19:746–60.
[18] ITSAC Consulting Group, Ltd. FLAC manuals, 1993.
[19] Granet S, Fabrie P, Lemonnier P, Quintard M. A two-phase flow simulation of a fractured reservoir using a new fissure element method. Journal of Petroleum Science and Engineering 2001;32(1):35–52.
[20] Caillabet Y, Fabrie P, Landerau P, Noetinger B, Quintard M. Implementation of a finite-volume method for the determination of effective parameters in fissured porous media. Numer Methods Partial Differential Equations 2000;16:237–63.
[21] Fang Z. A local degradation approach to the numerical analysis of brittle fracture in heterogeneous rocks. PhD thesis, Imperial College of Science, Technology and medicine, University of London, UK, 2001.
[22] Martino S, Prestininzi A, Scarascia Mugnozza G. Mechanisms of deep seated gravitational deformations: parameters from laboratory testing for analogical and numerical modeling. In: Särkkä, Eloranta, editors. Rock mechanics—a challenge for society. Swetz and Zeitlinger Lisse, ISBN 90 2651 821 B. 2001. p. 137–42.
[23] Kourdey A, Alheib M, Piguet JP. Evaluation of the slope stability by numerical methods. In: Särkkä, Eloranta, editors. Rock mechanics—a challenge for society. Swetz and Zeitlinger Lisse, ISBN 90 2651 821 B. 2001. p. 499–504.
[24] Marmo BA, Wilson CJL. A verification procedure for the use of FLAC to study glacial dynamics and the implementation of an anisotropic flow law. In: Särkkä, Eloranta, editors. Rock mechanics—a challenge for society. Swetz and Zeitlinger Lisse, ISBN 90 2651 821 B. 2001. p. 183–9.
[25] Mishev ID. Finite volume methods on Voronoi meshes. Numer Methods Partial Differential Equations 1998;14:193–212.
[26] Detournay C, Hart R. FLAC and numerical modelling in geomechanics. Proceedings of the International FLAC symposium on Numerical Modelling in Geomechanics, Minneapolis. Rotterdam: Balkema, 1999.
[27] Benito JJ, Ureña F, Gavete L. Influence of several factors in the generalized finite difference method. Appl Math Modelling 2000;25:1039–53.
[28] Oñate E, Cervera M, Zienkiewicz OC. A finite volume formulation for structural mechanics. Int J Numer Methods Eng 1994;37:181–201.
[29] Lahrmann A. An element formulation for the classical finite difference and finite volume method applied to arbitrarily shaped domains. Int J Numer Methods Eng 1992;35:893–913.
[30] Demirdžič I, Muzaferija S. Finite volume method for stress analysis in complex domains. Int J Numer Methods Eng 1994;37:3751–66.
[31] Demirdžič I, Horman I, Martinovič D. Finite volume analysis of stress and deformation in hydro-thermo-elastic orthotropic body. Comput Methods Appl Mech Eng 2000; 190:1221–32.
[32] Jasak H, Weller HG. Application of the finite volume method and unstructured meshes to linear elasticity. Int J Numer Methods Eng 2000;48:267–87.
[33] Cocchi GM. The finite difference method with arbitrary grids in the elastic–static analysis of three-dimensional continua. Comput Struct 2000;75:187–208.
[34] Courant R. Variational methods for the solution of problems of equilibrium and vibration. Bull Am Math Soc 1943;49:1–43.
[35] Prager W, Synge JL. Approximation in elasticity based on the concept of function space. Q J Appl Math 1947;5:214–69.
[36] Turner M, Clough RW, Martin HC, Topp LJ. Stiffness and deflection analysis of complex structures. J Aeronaut Sci 1956;23(9):805–23.
[37] Clough RW. The finite element method in plane stress analysis. Proceedings of the Second ASCE Conference Electronic Computation, Pittsburg, PA, 1960.
[38] Argyris J. Energy theorems and structural analysis. Aircraft engineering, 1954 and 1955. London: Reprinted by Butterworths Scientific Publications, 1960.
[39] Zienkiewizc OC. The finite element method in engineering sciences, 3rd ed. New York: McGraw-Hill, 1977.
[40] Bathe KJ. The finite element procedures in engineering analysis. Englewood Cliffs, NJ: Prentice-Hall, 1982.
[41] Beer G, Meck JL. Infinite domain elements. Int J Numer Methods Eng 1981;17(1):43–52.
[42] Zienkiewizc OC, Emson C, Bettess P. A novel boundary infinite element. Int J Num Meth Eng 1983;19:393–404.
[43] Bettes P. Infinite elements. Int J Numer Methods Eng 1977; 11:53–64.
[44] Cheng YM. The use of infinite element. Comput Geomech 1996;18(1):65–70.
[45] Owen DRJ, Hinton E. Finite elements in plasticity: theory and applications. Swansea, UK: Pineridge Press, 1980.
[46] Naylor DJ, Pande GN, Simpson B, Tabb R. Finite elements in geotechnical engineering. Swansea, UK: Pineridge Press, 1981.
[47] Pande GN, Beer G, Williams JR. Numerical methods in rock mechanics. New York: Wíley, 1990.
[48] Wittke W. Rock mechanics—theory and applications. Berlin: Springer, 1990.
[49] Beer G, Watson JO. Introduction to finite boundary element method for engineers. New York: John and Wiley, 1992.
[50] Tang C, Fu YF, Kou SQ, Lindqvist PA. Numerical simulation of loading in inhomogeneous rocks. Int J Rock Mech Min Sci 1998;35(7):1001–7.
[51] Goodman RE, Taylor RL, Brekke TL. A model for the mechanics of jointed rock. J Soil Mech Div ASCE 94, SM3, 1968. p. 637–59.
[52] Goodman RE. Methods of geological engineering in discontinuous rocks. San Francisco: West Publishing Company, 1976.
[53] Zienkiewicz OC, Best B, Dullage C, Stagg K. Analysis of non-linear problems in rock mechanics with particular reference to jointed rock systems. Proceedings of the Second International Congress on Rock Mechanics, Belgrade, 1970.
[54] Ghaboussi J, Wilson EL, Isenberg J. Finite element for rock joints and interfaces. J Soil Mech Div ASCE 99, SM10, 1973. p. 833–48.
[55] Katona MG. A simple contact–friction interface element with applications to buried culverts. Int J Numer Anal Methods Geomech 1983;7:371–84.
[56] Desai CS, Zamman MM, Lightner JG, Siriwardane HJ. Thin-layer element for interfaces and joints. Int J Numer Anal Methods Geomech 1984;8:19–43.
[57] Wang G, Yuan J. A new method for solving the contact–friction problem. In: Yuan J, editor. Computer methods and advances in geomechanics, vol. 2. Rotterdam: Balkema, 1997. p. 1965–7.
[58] Gens A, Carol I, Alonso EE. An interface element formulation for the analysis of soil–reinforcement interaction. Comput Geotech 1989;7:133–51.
[59] Gens A, Carol I, Alonso EE. Rock joints: fem implementation and applications. In: Selvadurai APS, Boulon M, editors. Mechanics of geomaterial interfaces. Amsterdam: Elsevier, 1995. p. 395–420.
[60] Buczkowski R, Kleiber M. Elasto-plastic interface model for 3D-frictional orthotropic contact problems. Int J Numer Methods Eng 1997;40:599–619.

[61] Wan RC. The numerical modeling of shear bands in geological materials. PhD thesis, University of Alberta, Edmonton, Alta., 1990.

[62] Belytschko T, Black T. Elastic crack growth in finite elements with minimal re-meshing. Int J Numer Methods Eng 1999; 45:601–20.

[63] Belytschko T, Moës N, Usui S, Parimi C. Arbitrary discontinuities in finite elements. Int J Numer Methods Eng 2001;50: 993–1013.

[64] Daux C, Moës N, Dolbow J, Sukumar N, Belytschko T. Arbitrary branched and intersecting cracks with the extended finite element method. Int J Numer Methods Eng 2000;48: 1741–60.

[65] Duarte CA, Babuška I, Oden JT. Generalized finite element methods for three-dimensional structural mechanics problems. Comput Struct 2000;77:215–32.

[66] Duarte CA, Hamzeh ON, Liszka TJ, Tworzydlo WW. A generalized finite element method for the simulation of three-dimensional dynamic crack propagation. Comput Methods Appl Mech Eng 2001;190:2227–62.

[67] Dolbow J, Moës N, Belytschko T. Discontinuous enrichment in finite elements with a partition of unity method. Finite Element Anal Design 2000;36:235–60.

[68] Jirasek M, Zimmermann T. Embedded crack model: I. Basic formulation. Int J Numer Methods Eng 2001;50:1269–90.

[69] Jirasek M, Zimmermann T. Embedded crack model: II: Combination with smeared cracks. Int J Numer Methods Eng 2001;50:1291–305.

[70] Moës N, Dolbow J, Belytschko T. A finite element method for crack growth without remeshing. Int J Numer Methods Eng 1999;46:131–50.

[71] Sukumar N, Moës N, Moran B, Belytschko T. Extended finite element method for three-dimensional crack modeling. Int J Numer Methods Eng 2000;48:1549–70.

[72] Stolarska M, Chopp DL, Moës N, Belytschko T. Modeling crack growth by level sets in the extended finite element method. Int J Numer Methods Eng 2001;51:943–60.

[73] Duarte AVC, Rochinha FA, do Carmo EDG. Discontinuous finite element formulations applied to cracked elastic domains. Comput Methods Appl Mech Eng 2000;185:21–36.

[74] Strouboulis T, Babuška I, Copps K. The design and analysis of the generalized finite element method. Comput Methods Appl Mech Eng 2000;181:43–69.

[75] Strouboulis T, Copps K, Banuška I. The generalized finite element method. Comput Methods Appl Mech Eng 2001;190: 4081–193.

[76] Shi G. Manifold method of material analysis. Transaction of the Ninth Army Conference on Applied Mathematics and Computing, Minneapolis, MN, 1991. p. 57–76.

[77] Shi G. Modeling rock joints and blocks by manifold method. Proceedings of the 33rd US Symposium on Rock Mechanics, Santa Fe, NM, 1992. p. 639–48.

[78] Chen G, Ohnishi Y, Ito T. Development of high-order manifold method. Int J Numer Methods Eng 1998;43:685–712.

[79] Wang Z, Wang S, Yang Z. Manifold method in analysis of large deformation for rock. Chin J Rock Mech Eng 1997; 16(5):399–404.

[80] Wang S, Ge X. Application of manifold method in simulating crack propagation. Chin J Rock Mech Eng 1997;16(5):405–10.

[81] Li C, Wang CY, Sheng J, editors. Proceedings of the First International Conference on Analysis of Discontinuous Deformation (ICADD-1), National Central University, Chungli, Taiwan, 1995.

[82] Salami MR, Banks D, editors. Discontinuous deformation analysis (DDA) and simulations of discontinuous media. Albuquerque, NM: TSI Press, 1996.

[83] Ohnishi Y, editor. Proceedings of the Second International Conferences of Discontinuous Deformation (ICADD-II), Kyoto, 1997.

[84] Amadei B, editor. Proceedings of the Third International Conferences of Discontinuous Deformation (ICADD-III), Vail, CO, 1999.

[85] Arnold DN. Discretization by finite elements of a model parameter dependent problem. Numer Math 1981;37:405–21.

[86] Babuška I, Suri M. On the locking and robustness in the finite element method. SIAM J Numer Anal 1992;29:1261–93.

[87] Suri M. Analytical and computational assessment of locking in the hp finite element method. Comput Methods Appl Mech Eng 1996;133:347–71.

[88] Bucalen M, Bathe KJ. Locking behaviour of isoparametric curved beam finite elements. Appl Mech Rev 1995;48/11/2: S25–9.

[89] Babuška I, Suri M. The p and h–p versions of the finite element method, an overview. Comput Methods Appl Mech Eng 1990;80:5–26.

[90] Chilton L, Suri M. On the selection of a locking-free hp element for elasticity problems. Int J Numer Methods Eng 1997;40: 2045–62.

[91] Oden JT. The best FEM. J Finite Element Anal Design 1990; 7(2):103–14.

[92] Belytchko T, Krongauz Y, Organ D, Fleming M, Krysl P. Meshless methods: an overview and recent developments. Comput Methods Appl Mech Eng 1996;139:3–47.

[93] Monaghna JJ. An introduction to SPH. Comput Phys Commun 1988;48:89–96.

[94] Randles PW, Libersky LD. Smoothed particle hydrodynamics: some recent improvements and applications. Comput Methods Appl Mech Eng 1996;139:375–408.

[95] Nayroles B, Touzot G, Villon P. Generalizing the finite element method: diffuse approximation and diffuse elements. Comput Mech 1992;10:307–18.

[96] Belytschko T, Lu YY, Gu L. Element-free Galerkin method. Int J Numer Methods Eng 1994;37:229–56.

[97] Liu KW, Jun S, Zhnag YF. Reproducing kernel particle methods. Int J Numer Eng 1995;20:1081–106.

[98] Liu KW, Chen Y, Uras RA, Chang CT. Generalized multiple scale reproducing kernel particle methods. Comput Methods Appl Mech Eng 1996;139:91–157.

[99] Chen JS, Pan C, Wu CT, Liu WK. Reproducing kernel particle methods for large deformation analysis of non-linear structures. Comput Methods Appl Mech Eng 1996;139:195–227.

[100] Liu WK, Li S, Belytschko T. Moving least-square reproducing kernel methods, Part I: methodology and convergence. Comput Methods Appl Mech Eng 1997;143:113–54.

[101] Duarte CA, Oden JT. H–p clouds—an hp-meshless method. Numer Methods Partial Differential Equations 1996;12:673–705.

[102] Liszka TJ, Durate CA, Twozydlo WW. Hp-meshless cloud method. Comput Methods Appl Mech Eng 1996;139:263–88.

[103] Melenk JM, Babuška I. The partition of unity finite element method: basic theory and applications. Comput Methods Appl Mech Eng 1996;139:289–314.

[104] Atluri SN, Zhu T. A new meshless local Petrov–Galerkin (MLPG) approach in computational mechanics. Comput Mech 1998;22:117–27.

[105] Atluri SN, Kim HG, Cho JY. A critical assessment of the truly local Petrov–Galerkin (MLPG) and local boundary integral equation (LBIE) methods. Comput Mech 1999;24:348–72.

[106] De S, Bathe KJ. The method of finite spheres. Comput Mech 2000;25:329–45.

[107] Oñate E, Idelsohn S, Zienkiewicz OC, Taylor RL, Sacco C. Stabilized finite point method for analysis of fluid mechanics problems. Comput Methods Appl Mech Eng 1996;139:315–46.

[108] Sulsky D, Schreye HL. Axisymmetric form of the material point method with applications to upsetting and Taylor impact problems. Comput Methods Appl Mech Eng 1996;139: 409–29.
[109] Sukumar N, Moran B, Belytschko T. The natural element method in solid mechanics. Int J Numer Methods Eng 1998; 43:839–87.
[110] Krongauz Y, Belytschko T. Enforcement of essential boundary conditions in meshless approximations using finite elements. Comput Methods Appl Mech Eng 1996;131:133–45.
[111] Zhang X, Liu XH, Song KZ, Lu MW. Least-square collocation meshless method. Int J Numer Methods Eng 2001;51:1089–100.
[112] Atluri SN, Li G. Finite cloud method: a true meshless technique based on a fixed reproducing kernel approximation. Int J Numer Methods Eng 2001;50:2373–410.
[113] Zhang X, Lu M, Wegner JL. A 2-D meshless model for jointed rock structures. Int J Numer Methods Eng 2000;47:1649–61.
[114] Belytschko T, Organ D, Gerlach C. Element-free Galerkin methods for dynamic fracture in concrete. Comput Methods Appl Mech Eng, 2000;187:385–99.
[115] Li S, Qian D, Liu WK, Belytschko T. A meshfree contact-detection algorithm. Comput Methods Appl Mech Eng 2001; 190:3271–92.
[116] Fratantonio M, Rencis JJ. Exact boundary element integrations for two-dimensional Laplace equation. Eng Anal Boundary Elements 2000;24:325–42.
[117] Carini A, Diligenti M, Maranesi P, Zanella M. Analytical Integration for two-dimensional elastic analysis by the symmetric Galerkin boundary element method. Comput Mech 1999;23:308–23.
[118] Jaswon MA. Integral equation methods in potential theory. I. Proc R Soc London A 1963;275:23–32.
[119] Symm GT. Integral equation method in potential theory—II. Proc R Soc London A 1963;275:33–46.
[120] Rizzo FJ. An integral equation approach to boundary value problems of classical elastostatics. Q Appl Math 1967;25:83–95.
[121] Cruse TA, Rizzo J. A direct formulation and numerical solution of the general transient elastodynamic problems. Int J Math Anal Appl 1968;22:244–59.
[122] Cruse TA. Application of the boundary integral equation method for three-dimensional stress analysis. Comput Struct 1973;3:509–27.
[123] Cruse TA. Two-dimensional BEM fracture mechanics analysis. Appl Math Modelling 1978;2:287–93.
[124] Betti E.Teoria dell Elastocita, II Nuovo Cimento (Ser. 2), 7 and 8, 1872.
[125] Somigliana C. Sopra L'equilibro di un corpo elastico isotropo, II Nuvo Cimento (Ser. 3), tt, 17-20, 1885.
[126] Brebbia CA, Dominguez J. Boundary element method for potential problems. J Appl Math Modelling 1977;1:372–78; London: Metallurgy Publication.
[127] Lachat JC, Watson JO. Effective numerical treatment of boundary integral equations: a formulation for three-dimensional elastostatics. Int J Numer Methods Eng 1976;10: 991–1005.
[128] Watson JO. Advanced implementation of the boundary element method for two- and three-dimensional elastostatics. In: Banerjee PK, Butterfield R, editors. Developments in boundary element methods, vol. 1. London: Applied Science Publishers, 1979. p. 31–63.
[129] Crouch SL, Fairhurst C. Analysis of rock deformations due to excavation. Proc ASME Symp Rock Mech 1973;26–40.
[130] Brady BHG, Bray JW. The boundary element method for determining stress and displacements around long openings in a triaxial stress field. Int J Rock Mech Min Sci Geomech Abstr 1978;15:21–8.
[131] Crouch SL, Starfield AM. Boundary element methods in solid mechanics. London: George Allen & Unwin, 1983.
[132] Hoek E, Brown ET. Underground excavations in rock. London, UK: Institute of Mining and Metallurgy, 1982.
[133] Brebbia CA, editor. Topics in boundary element research, vol. 4. Applications in geomechanics. Berlin: Springer, 1987;
Venturini WS, Brebbia CA. Some applications of the boundary element method in geomechanics. Int J Numer Methods Eng 1983;7:419–43.
[134] Beer G, Pousen A. Efficient numerical modelling of faulted rock using the boundary element method. Int J Rock Mech Min Sci 1995;32(3):117A.
[135] Beer G, Pousen A. Rock joints-BEM computations. In: Sevaldurai, Boulon, editors. Mechanics of geomaterial interfaces. Rotterdam: Elsevier, 1995. p. 343–73.
[136] Kayupov MA, Kuriyagawa M. DDM modelling of narrow excavations and/or cracks in anisotropic rock mass. In: Barla G, editor. Proceedings of the Eurock '96. Rotterdam: Balkema, 1996. p. 351–8.
[137] Cerrolaza M, Garcia R. Boundary elements and damage mechanics to analyze excavations in rock mass. Eng Anal Boundary Elements 1997;20:1–16.
[138] Pan E, Amadei B, Kim YI. 2D BEM analysis of anisotropic half-plane problems—application to rock mechanics. Int J Rock Mech Min Sci 1998;35(1):69–74.
[139] Shou KJ. A three-dimensional hybrid boundary element method for non-linear analysis of a weak plane near an underground excavation. Tunnelling Underground Space Technol 2000;16(2): 215–26.
[140] Tian Y. A two-dimensional direct boundary integral method for elastodynamics. PhD thesis, University of Minnesota, Minneapolis, 1990.
[141] Siebrits E, Crouch SL. Geotechnical applications of a two-dimensional elastodynamic displacement discontinuity method. Int J Rock Mech Min Sci Geomech Abstr 1993; 30(7):1387–93.
[142] Birgisson B, Crouch SL. Elastodynamic boundary element method for piecewise homogeneous media. Int J Numer Methods Eng 1998;42(6):1045–69.
[143] Wang BL, Ma QC. Boundary element analysis methods for ground stress field of rock masses. Comput Geotech 1986;2: 261–74.
[144] Jing L. Characterization of jointed rock mass by boundary integral equation method. Proceedings of the International Symposium on Mining Technology and Science. Xuzhow, China: Trans Tech Publications, 1987. p. 794–9.
[145] Lafhaj Z, Shahrour I. Use of the boundary element method for the analysis of permeability tests in boreholes. Eng Anal Boundary Elements 2000;24:695–8.
[146] Pan E, Maier G. A symmetric integral approach to transient poroelastic analysis. Comput Mech 1997;19:169–78.
[147] Elzein AH. Heat sources in non-homogeneous rock media by boundary elements. Proceedings of the Geotechnical Engineering, Sydney, Australia, 2000.
[148] Ghassemi A, Cheng AHD, Diek A, Roegiers JC. A complete plane strain fictitious stress boundary element method for poroelastic media. Eng Anal Boundary Elements 2001;25: 41–8.
[149] Kuriyama K, Mizuta Y. Three-dimensional elastic analysis by the displacement discontinuity method with boundary division into triangle leaf elements. Int J Rock Mech Min Sci Geomech Abstr 1993;30(2):111–23.
[150] Kuriyama K, Mizuta Y, Mozumi H, Watanabe T. Three-dimensional elastic analysis by the boundary element method with analytical integrations over triangle leaf elements. Int J Rock Mech Min Sci Geomech Abstr 1995;32(1):77–83.

[151] Cayol Y, Cornet FH. 3D mixed boundary elements for elastostatic deformation field analysis. Int J Rock Mech Min Sci 1997;34(2):275–87.

[152] Crouch SL. Solution of plane elasticity problems by the displacement discontinuity method. Int J Numer Methods Eng 1976;10:301–43.

[153] Weaver J. Three dimensional crack analysis. Int J Solids Struct 1977;13:321–30.

[154] Dunbar WS. The equivalence between the displacement discontinuity and quadrupole boundary element methods. Int J Rock Mech Min Sci Geomech Abstr 1985;22(6):475–6.

[155] Blandford GE, Ingraffea AR, Ligget JA. Two-dimensional stress intensity factor computations using the boundary element method. Int J Numer Methods Eng 1981;17:387–406.

[156] Portela A. Dual boundary element incremental analysis of crack growth. PhD thesis, Wessex Institute of Technology, Portsmouth University, Southampton, UK, 1992.

[157] Portela A, Aliabadi MH, Rooke DP. The dual boundary element method: effective implementation for crack problems. Int J Numer Methods Eng 1992;33:1269–87.

[158] Portela A, Aliabadi MH, Rooke DP. Dual boundary element incremental analysis of crack propagation. Comput Struct 1993;46(2):237–84.

[159] Mi Y, Aliabadi MH. Dual boundary element method for three-dimensional fracture mechanics analysis. Eng Anal Boundary Elements 1992;10:161–71.

[160] Mi Y, Aliabadi MH. Three-dimensional crack growth simulation using BEM. Comput Struct 1994;52:871–8.

[161] Yamada Y, Ezawa Y, Nishiguchi I. Reconsiderations on singularity or crack tip elements. Int J Numer Methods Eng 1979;14:1525–44.

[162] Aliabadi MH, Rooke DP. Numerical fracture mechanics. Southampton: Computational Mechanics Publications; Dordrecht: Kluwer Academic Publishers, 1991.

[163] Wen PH, Wang Y. The calculation of SIF considering the effects of arc crack surface contact and friction under uniaxial tension and pressure. Eng Fract Mech 1991;39:651–60.

[164] Shen B. Mechanics of fractures and intervening bridges in hard rocks. PhD thesis, Royal Institute of Technology, Stockholm, Sweden, 1991.

[165] Wen PH. Dynamic fracture mechanics: displacement discontinuity method. Southampton: Computational Mechanics Publishers, 1996.

[166] Mi Y. Three-dimensional analysis of crack growth. Southampton: Computational Mechanics Publications, 1996.

[167] Aliabadi MH, editor. Fracture of rock. Boston, Southampton: WIT press, Computational Mechanics Publications, 1999.

[168] Tan XC, Kou SQ, Lindqvist PA. Application of the DDM and fracture mechanics model on the simulation of rock breakage by mechanical tools. Eng Geol 1998;49(4):277–84.

[169] La Pointe PR, Cladouhos T, Follin S. Calculation of displacement on fractures intersecting canisters induced by earthquakes: Aberg, Beberg and Ceberg examples. Technical Report of Swedish Nuclear Fuel and Waste Management Co. (SKB), SKB TR-99-03, 1999.

[170] Crotty JM, Wardle LJ. Boundary integral analysis of piecewise homogeneous media with structural discontinuities. Int J Rock Mech Min Sci Geomech Abstr 1985;22(6):419–27.

[171] Bonnet M, Maier G, Polizzotto C. Symmetric Galerkin boundary element methods. Appl Mech Rev 1998;51(11):669–704.

[172] Salvadori A. Analytical integration of hypersingular kernel in 3D BEM problems. Comput Methods Appl Mech Eng 2001;190:3957–75.

[173] Michael O, Barbon PE. Galerkin formulation and singularity subtraction for spectral solutions of boundary integral equations. Int J Numer Methods Eng 1998;41:95–111.

[174] Wang J, Mogilevskaya SG, Crouch SL. A Galerkin boundary integral method for non-homogenous materials with crack. In: Elsworth D, Tinucci JP, Heasley KA, editors. Rock mechanics in the national interest. Rotterdam: Balkema, 2001. p. 1453–60.

[175] Nagarajan A, Lutz E, Mukherjee S. A novel boundary element method for linear elasticity with no numerical integration for two-dimensional and line integrals for the three-dimensional problems. J Appl Mech Trans ASME Appl Mech Div 1994; 61:264–9.

[176] Nagarajan A, Mukherjee S, Lutz E. The boundary contour method for three-dimensional linear elasticity. J Appl Mech Trans ASME Appl Mech Div 1996;63:278–86.

[177] Phan AV, Mukherjee S, Mayer JRR. The boundary contour method for two-dimensional linear elasticity with quadratic boundary elements. Comput Mech 1997;20:310–9.

[178] Mukherjee YX, Mukherjee S, Shi X, Nagarajan A. The boundary contour method for three-dimensional linear elasticity with a new quadratic boundary elements. Eng Anal Boundary Elements 1997;20:35–44.

[179] Zhou S, Cao Z, Sun S. The traction boundary contour method for linear elasticity. Int J Numer Methods Eng 1999;46: 1883–95.

[180] Zhou S, Sun S, Cao Z. The dual boundary contour method for two-dimensional crack problems. Int J Fract 1998;92:201–12.

[181] Novati G, Springhetti R. A Galerkin boundary contour method for two-dimensional linear elasticity. Comput Mech 1999;23:53–62.

[182] Chati MK, Paulino GH, Mukherjee S. The meshless standard and hypersingular boundary node methods—applications to error estimation and adaptivity in three-dimensional problems. Int J Numer Methods Eng 2001;50:2233–69.

[183] Mukherjee, YX, Mukherjee S. The boundary node method for potential problems. Int J Numer Methods Eng 1997;40: 797–815.

[184] Chati MK, Mukherjee S, Mukherjee YX. The boundary node method for three-dimensional linear elasticity. Int J Numer Methods Eng 1999;46:1163–84.

[185] Chati MK, Mukherjee S. The boundary node method for three-dimensional problems in potential theory. Int J Numer Methods Eng 2000;47:1523–47.

[186] Kothnur VS, Mukherjee S, Mukherjee YX. Two-dimensional linear elasticity by the boundary node method. Int J Solids Struct 1999;36:1129–47.

[187] Gu YT, Liu GR. A boundary point interpolation method for stress analysis of solids. Comput Mech 2002;28:47–54.

[188] Gowrishankar R, Mukherjee S. A 'pure' boundary node method for potential problems. Commun Numer Meth Eng 2002;18: 411–27.

[189] Brebbia CA, Telles JCF, Wrobel LC. Boundary element techniques: theory and applications in engineering. Berlin: Springer, 1984.

[190] Partridge PW, Brebbia CA, Wrobel LC. The dual reciprocity boundary element method. Southampton and Boston: Computational Mechanics Publications and Elsevier, 1992.

[191] Cheng AHD, Lafe O, Grilli S. Dual-reciprocity BEM based on global interpolation functions. Eng Anal Boundary Elements 1994;13:303–11.

[192] Cheng AHD, Chen CS, Golberg MA, Rashed YF. BEM for thermoelasticity and elasticity with body force—a revisit. Eng Anal Boundary Elements 2001;25:377–87.

[193] El Harrouni K, Ouazar D, Wrobel LC, Cheng AHD. Groundwater parameter estimation by optimization and DRBEM. Eng Anal Boundary Elements 1997;19:97–103.

[194] Ochiai Y, Kobayashi T. Initial stress formulation for elastoplastic analysis by improved multi-reciprocity boundary element method. Eng Anal Boundary Elements 1999;23:167–73.

[195] Ochiai Y, Kobayashi T. Initial strain formulation without internal cells for elastoplastic analysis by triple-reciprocity BEM. Int J Numer Methods Eng 2001;50:1877–92.
[196] Gao XW. A boundary element method without internal cells for two-dimensional and three-dimensional elastoplastic problems. J Appl Mech Trans ASME Appl Mech Div 2002;69: 154–60.
[197] Burman BC. A numerical approach to the mechanics of discontinua. PhD thesis, James Cook University of North Queensland, Townsville, Australia, 1971.
[198] Cundall PA. A computer model for simulating progressive, large scale movements in blocky rock systems. Proceedings of the International Symposium Rock Fracture, ISRM, Nancy, Paper No. II-8, vol. 1, 1971.
[199] Cundall PA. Rational design of tunnel supports: a computer model for rock mass behaviour using interactive graphics for the input and output of geomaterial data. Technical Report MRD-2-74, Missouri River Division, US Army Corps of Engineers, NTIS Report No. AD/A-001 602, 1974.
[200] Chappel BA. The mechanics of blocky material. PhD thesis, Australia National University, Canberra, 1972.
[201] Chappel BA. Numerical and physical experiments with discontinua. Proceedings of the Third Congress on ISRM, vol. 2A, Denver, CO., 1974.
[202] Byrne RJ. Physical and numerical model in rock and soil-slope stability. PhD thesis, James Cook University of North Queensland, Townsville, Australia, 1974.
[203] Lin D, Fairhurst C, Starfield AM. Geometrical identification of three-dimensional rock block systems using topological techniques. Int J Rock Mech Min Sci Geomech Abstr 1987;24(6): 331–8.
[204] Lin D, Fairhurst C. Static analysis of the stability of three-dimensional blocks systems around excavations in rock. Int J Rock Mech Min Sci Geomech Abstr 1988;25(3):139–47.
[205] Lin D. Elements of rock block modelling. PhD thesis, University of Minnesota, Minneapolis, 1992.
[206] Jing L, Stephansson O. Topological identification of block assemblage for jointed rock masses. Int J Rock Mech Min Sci Geomech Abstr 1994;31(2):163–72.
[207] Jing L, Stephansson O. Identification of block topology for jointed rock masses using boundary operators. Proceedings of the International ISRM Symposium on Rock Mechanics, Santiago, Chile, vol. I, 1994. p. 19–29.
[208] Jing L. Block system construction for three-dimensional discrete element models of fractured rocks. Int J Rock Mech Min Sci 2000;37(4):645–59.
[209] Ju J. Systematic identification of polyhedral blocks with arbitrary joints and faults. Comput Geotech 2002;29:49–72.
[210] Cundall PA. UDEC—a generalized distinct element program for modelling jointed rock. Report PCAR-1-80, Peter Cundall Associates, European Research Office, US Army Corps of Engineers, 1980.
[211] Cundall PA. Formulation of a three-dimensional distinct element model—Part I: a scheme to detect and represent contacts in a system composed of many polyhedral blocks. Int J Rock Mech Min Sci Geomech Abstr 1988;25(3):107–16.
[212] ITASCA Consulting Group, Inc. UDEC Manual, 1992.
[213] ITASCA Consulting Group, Inc. 3DEC Manual, 1994.
[214] Taylor LM. BLOCKS: a block motion code for geomechanics studies. SANDIA Report SANDD82-2373, Sandia National Laboratories, 1983.
[215] Williams JR, Hocking G, Mustoe GGW. The theoretical basis of the discrete element method. NUMETA '85, Numerical Methods in Engineering, Theory and Application, Conference in Swansea, January 7–11. Rotterdam: A.A. Balkema Publishers, 1985. p. 897–906.
[216] Williams JR, Mustoe GGW. Modal methods for the analysis of discrete systems. Comput Geotech 1987;4:1–19.
[217] Williams JR. Contact analysis of large number of interacting bodies using the discrete model methods for simulating material failure on microscopic scale. Eng Comput 1988;5:198–209.
[218] Willaims JR, Pentland AP. Superquadrics and modal dynamics for discrete elements in interactive design. Eng Comput 1992;9:115–27.
[219] Mustoe GGW. A generalized formulation of the discrete element method. Eng Comput 1992;9:181–90.
[220] Hocking G. Development and application of the boundary integral and rigid block method for geotechnics. PhD thesis, University of London, 1977.
[221] Hocking G. The discrete element method for analysis of fragmentation of discontinua. Eng Comput 1992;9:145–55.
[222] Williams JR, O'Connor R. A linear complexity intersection algorithm for discrete element simulation of arbitrary geometries. Eng Comput 1995;12:185–201.
[223] Kawai T. New discrete structural models and generalization of the method of limit analysis. Proceedings of the International Conference on Finite Elements Nonlinear Solid Struct Mech, Norway, 1977;2:G04.1–G04.20.
[224] Kawai T. New element models in discrete structural analysis. Jpn Soc Naval Arch 1977;141:174–80.
[225] Kawai T, Kawabata KY, Kondou I, Kumagai K. A new discrete model for analysis of solid mechanics problems. Proceedings of the First Conference Numerical Methods in Fracture Mechanics, Swansea, UK, 1978. p. 26–7.
[226] Wang B, Garga VK. A numerical method for modeling large displacements of jointed rocks—Part I: fundamentals. Can Geotech J 1993;30:96–108.
[227] Wang B, Vongpaisal S, Dunne K, Pakalnis R. Prediction and measurement of hangingwall movements of Detour Lake mine SLR stope. In: Yuan J, editor. Computer methods and advances in geomechanics, vol. 2. Rotterdam: Balkema, 1997. p. 1571–5.
[228] Li G, Vance J. A 3-D block-spring model for simulating the behaviour of jointed rocks. In: Amadei B, Kranz RL, Scott GA, Smeallie PH, editors. Rock mechanics for industry, vol. 1. Rotterdam: Balkema, 1999. p. 141–6.
[229] Hu Y. Block-spring-model considering large displacements and non-linear stress–strain relationships of rock joints. In: Yuan J, editor. Computer methods and advances in geomechanics, vol. 1. Rotterdam: Balkema, 1997. p. 507–12.
[230] Li G, Wang B. Development of a 3-D block-spring model for jointed rocks. In: Rossmanith HP, editor. Mechanics of jointed and faulted rock. Rotterdam: Balkema, 1998. p. 305–9.
[231] Shi G. Discontinuous deformation analysis—a new numerical model for the statics, dynamics of block systems. PhD thesis, University of California, Berkeley, USA, 1988.
[232] Shyu K. Nodal-based discontinuous deformation analysis. PhD thesis, University of California, Berkeley, 1993.
[233] Chang QT. Nonlinear dynamic discontinuous deformation analysis with finite element meshed block systems. PhD thesis, University of California, Berkeley, 1994.
[234] Kim Y, Amadei B, Pan E. Modeling the effect of water, excavation sequence and rock reinforcement with discontinuous deformation analysis. Int J Rock Mech Min Sci 1999;36(7): 949–70.
[235] Jing L, Ma Y, Fang Z. Modelling of fluid flow and solid deformation for fractured rocks with discontinuous deformation analysis (DDA) method. Int J Rock Mech Min Sci 2001;38(3): 343–55.
[236] Ghaboussi J. Fully deformable discrete element analysis using a finite element approach. Int J Comput Geotech 1988;5: 175–95.

[237] Barbosa R, Ghaboussi J. Discrete finite element method. Proceedings of the First US Conference on discrete element methods, Golden, CO, 1989.

[238] Barbosa R, Ghaboussi J. Discrete finite element method for multiple deformable bodies. J Finite Elements Anal Design 1990;7:145–58.

[239] Munjiza A, Owen DRJ, Bicanic N. A combined finite-discrete element method in transient dynamics of fracturing solid. Int J Eng Comput 1995;12:145–74.

[240] Munjiza A, Andrews KRF, White JK. Combined single and smeared crack model in combined finite-discrete element analysis. Int J Numer Methods Eng 1999;44:41–57.

[241] Munjiza A, Andrews KRF. Penalty function method for combined finite-discrete element systems comprising large number of separate bodies. Int J Numer Methods Eng 2000; 49:1377–96.

[242] Cundall PA, Marti J. Some new developments in discrete numerical methods for dynamic modelling of jointed rock masses. Proc Conf Rapid Excavation Tunnelling 1979;2: 1464–6.

[243] Cundall PA, Hart RD. Development of generalized 2-D and 3-D distinct element programs for modelling jointed rock. ITASCA Consulting Group, Misc. Paper No. SL-85-1. U.S. Army Corps of Engineers, 1985.

[244] Lemos JV. A hybrid distinct element computational model for the half-plane. MSc thesis, Department of Civil Engineering, University of Minnesota, 1987.

[245] Hart RD, Cundall PA, Lemos JV. Formulation of a three-dimensional distinct element method—Part II: mechanical calculations for motion and interaction of a system composed of many polyhedral blocks. Int J Rock Mech Min Sci Geomech Abstr 1988;25(3):117–25.

[246] Cundall PA, Hart RD. Numerical modelling of discontinua. Eng Comput 1992;9:101–13.

[247] Hart RD. An introduction to distinct element modelling for rock engineering. In: Hudson JA, editor in-chief. Comprehensive rock engineering, vol. 2. Oxford: Pergamon Press, 1993. p. 245–61.

[248] Curran JH, Ofoegbu GI. Modeling discontinuities in numerical analysis. In: Hudson JA, editor in-chief. Comprehensive rock engineering, vol. 1. Oxford: Pergamon Press, 1993. p. 443–68.

[249] Southwell RV. Stress calculation in frameworks by the method of systematic relaxation of constraints. Parts I and II. Proc Roy Soc (A) 1935;151:56–95.

[250] Southwell RV. Relaxation methods in engineering sciences—a treatise on approximate computation. Oxford: Oxford University Press, 1940.

[251] Southwell RV. Relaxation methods in theoretical physics—a continuation of the treatise: relaxation methods in engineering science, vol. I and II. Oxford: Oxford University Press, 1956.

[252] Barton N. Modeling jointed rock behaviour and tunnel performance. World Tunnelling 1991;4(7):414–6.

[253] Jing L, Stephansson O. Distinct element modelling of sublevel stoping. Proceedings of the Seventh International Congress of ISRM, Aachen, Germany, vol. 1, 1991. p. 741–6.

[254] Nordlund E, Rådberg G, Jing L. Determination of failure modes in jointed pillars by numerical modelling. In: Myer LR, Cook NGW, Goodman RE, Tsang CF, editors. Fractured and jointed rock masses. Rotterdam: Balkema, 1995. p. 345–50.

[255] Chryssanthakis P, Barton N, Lorig L, Christiansson M. Numerical simulation of fibre reinforced shotcrete in a tunnel using the distinct element method. Int J Rock Mech Min Sci 1997;34(3–4), Paper No. 054.

[256] Hanssen FH, Spinnler L, Fine J. A new approach for rock mass capability. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1379–85.

[257] Kochen R, Andrade JCO. Predicted behaviour of a subway station in weathered rock. Int J Rock Mech Min Sci 1997;34 (3–4), Paper No. 160.

[258] McNearny RL, Abel JF. Large-scale two-dimensional block caving model tests. Int J Rock Mech Min Sci Geomech Abstr 1993;30(2):93–109.

[259] Souley M, Hommand F, Thoraval A. The effect of joint constitutive laws on the modelling of an underground excavation and comparison with in-situ measurements. Int J Rock Mech Min Sci 1997;34(1):97–115.

[260] Souley M, Hoxha D, Hommand F. The distinct element modelling of an underground excavation using a continuum damage model. Int J Rock Mech Min Sci 1997;35(4–5), Paper No. 026.

[261] Sofianos AI, Kapenis AP. Numerical evaluation of the response in bending of an underground hard rock voussoir beam roof. J Rock Mech Min Sci 1998;35(8):1071–86.

[262] Lorig L, Gibson W, Alvial J, Cuevas J. Gravity flow simulations with the particle flow code (PFC). ISRM News J 1995;3(1): 18–24.

[263] Zhao J, Zhou YX, Hefny AM, Cai JG, Chen SG, Li HB, Liu JF, Jain M, Foo ST, Seah CC. Rock dynamics research related to cavern development for ammunition storage. Tunnelling Underground Space Technol 1999;14(4):513–26.

[264] Cai JG, Zhao J. Effects of multiple parallel fractures on apparent attenuation of stress waves in rock mass. Int J Rock Mech Min Sci 2000;37:661–82.

[265] Chan T, Khair K, Jing L, Ahola M, Noorishad J, Vuillold E. International comparison of coupled thermo-hydro-mechanical models of a multiple-fracture bench mark problem: DECOVALEX Phase I, Bench Mark Test 2. Int J Rock Mech Min Sci Geomech Abstr 1995;32(5):435–52.

[266] Hansson H, Jing L, Stephansson O. 3-D DEM modelling of coupled thermo-mechanical response for a hypothetical nuclear waste repository. Proceedings of the NUMOG V—International Symposium on Numerical Models in Geomechanics, Davos, Switzerland. Rotterdam: Balkema, 1995. p. 257–62.

[267] Jing L, Tsang CF, Stephansson O. DECOVALEX—an international co-operative research project on mathematical models of coupled THM processes for safety analysis of radioactive waste repositories. Int J Rock Mech Min Sci Geomech Abstr 1995;32(5):389–98.

[268] Jing L, Hansson H, Stephansosn O, Shen B. 3D DEM study of thermo-mechanical responses of a nuclear waste repository in fractured rocks—far and near-field problems. In: Yuan J, editor. Computer methods and advances in geomechanics, vol. 2. Rotterdam: Balkema, 1997. p. 1207–14.

[269] Gutierrez M, Makurat A. Coupled THM modelling of cold water injection in fractured hydrocarbon reservoirs. Int J Rock Mech Min Sci 1997;34(3–4), Paper No. 113.

[270] Harper TR, Last NC. Interpretation by numerical modeling of changes of fracture system hydraulic conductivity induced by fluid injection. Géotechnique 1989;39(1):1–11.

[271] Harper TR, Last NC. Response of fractured rocks subjected to fluid injection. Part II: characteristic behaviour. Techtonophysics 1990;172:33–51.

[272] Harper TR, Last NC. Response of fractured rocks subjected to fluid injection. Part III: practical applications. Techtonophysics 1990;172:53–65.

[273] Zhu W, Zhang Q, Jing L. Stability analysis of the ship-lock slopes of the Three-Gorge project by three-dimensional FEM and DEM techniques. In: Vail USA, Amadei B, editors. Proceedings of the Third International Conference of discontinuous deformation Analysis (ICADD-3). Alexandria, USA: American Rock Mechanics Association (ARMA), 1999. p. 263–72.

[274] Jing L, Stephansson O, Nordlund E. Study of rock joints under cyclic loading conditions. Rock Mech Rock Eng 1993;26(3): 215–32.
[275] Jing L, Nordlund E, Stephansson O. A 3-D constitutive model for rock joints with anisotropic friction and stress dependency in shear stiffness. Int J Rock Mech Min Sci Geomech Abstr 1994;31(2):173–8.
[276] Lanaro F, Jing L, Stephansson O, Barla G. DEM modelling of laboratory tests of block toppling. Int J Rock Mech Min Sci 1997;34(3–4):506–7.
[277] Makurat A, Ahola M, Khair K, Noorishad J, Rosengren L, Rutqvist J. The DECOVALEX test case one. Int J Rock Mech Min Sci Geomech Abstr 1995;32(5):399–408.
[278] Liao QH, Hencher SR. Numerical modeling of the hydro-mechanical behaviour of fractured rock masses. Int J Rock Mech Min Sci 1997;34(3–4), Paper No. 177.
[279] Lorig L. A simple numerical representation of fully bounded passive rock reinforcement for hard rock. Comput Geotech 1985;1:79–97.
[280] Jing L. Numerical modelling of jointed rock masses by distinct element method for two and three-dimensional problems. PhD thesis, 1990:90 D, Luleå University of Technology, Luleaå, Sweden, 1990.
[281] Rawlings CG, Barton NR, Bandis SC, Addis MA, Gutierrez MS. Laboratory and numerical discontinuum modeling of wellbore stability. J Pet Tech 1993;45(11):1086–92.
[282] Santarelli FJ, Dahen D, Baroudi H, Sliman KB. Mechanisms of borehole instability in heavily fractured rock. Int J Rock Mech Min Sci Geomech Abstr 1992;29(5):457–67.
[283] Hazzard JF, Young RP. Simulating acoustic emissions in bonded-particle models of rock. Int J Rock Mech Min Sci 2000;37(5):867–72.
[284] Zhang X, Sanderson DJ, Harkness RM, Last NC. Evaluation of the 2-D permeability tensor for fractured rock masses. Int J Rock Mech Min Sci Geomech Abstr 1996;33(1):17–37.
[285] Mas-Ivas D, Min KB, Jing L. Homogenization of mechanical properties of fracture rocks by DEM modeling. In: Wang S, Fu B, Li Z, editors. Frontiers of rock mechanics and sustainable development in 21st century. Proceedings of the Second Asia Rock Mechanics Symposium, September 11–14, 2001, Beijing, China. Rotterdam: Balkema, 2001. p. 311–4.
[286] Min KB, Mas-Ivars D, Jing L. Numerical derivation of the equivalent hydro-mechanical properties of fractured rock masses using distinct element method. In: Elsworth D, Tinucci JP, Heasley KA, editors. Rock mechanics in the national interest: Swets & Zeitlinger Lisse, 2001. p. 1469–76.
[287] Sharma VM, Saxena KR, Woods RD, editors. Distinct element modelling in geomechanics. Rotterdam: A.A. Balkema, 2001.
[288] Shi G, Goodman RE. Two dimensional discontinuous deformation analysis. Int J Numer Anal Methods Geomech 1985;9: 541–56.
[289] Jing L. Formulation of discontinuous deformation analysis (DDA)—an implicit discrete element model for block systems. Eng Geol 1998;49:371–81.
[290] Jing L. Contact formulations via energy minimization. Proceedings of the Second International Conference on DEM. In: Williams JR, Mustoe GGW, editors. Boston: IESL Publications, MIT, 1993. p. 15–26.
[291] Chen G, Miki S, Ohnishi Y. Practical improvement on DDA. In: Salami MR, Banks D, editors. Discontinuous deformation analysis (DDA) and simulations of discontinuous media. TSI Press, 1996. p. 302–09.
[292] Chen G, Ohnishi Y, Ito T. Development of high-order manifold method. Int J Numer Methods Eng 1998;43:685–712.
[293] Koo CY, Chern JC. Modification of the DDA method for rigid block problems. Int J Rock Mech Min Sci 1998;35(6):683–93.
[294] Lin CT, Amadei B, Jung J, Dwyer J. Extensions of discontinuous deformation analysis for jointed rock masses. Int J Rock Mech Min Sci Geomech Abstr 1996;33(7):671–94.
[295] Doolin DM, Sitar N. DDAML-discontinuous deformation analysis markup language. Int J Rock Mech Min Sci 2001;38: 467–74.
[296] Shi G. Three dimensional discontinuous deformation analysis. In: Elworth D, Tinucci JP, Heasley KA, editors. Rock mechanics in the national interest: Swets & Zeitlinger Lisse, 2001. p. 1421–8.
[297] Hsiung SM. Discontinuous deformation analysis (DDA) with n-th order polynomial displacement functions. In: Elworth D, Tinucci JP, Heasley KA, editors. Rock mechanics in the national interest: Swets & Zeitlinger Lisse, 2001. p. 1437–44.
[298] Zhang X, Lu MW. Block-interfaces model for non-linear numerical simulations of rock structures. Int J Rock Mech Min Sci 1998;35(7):983–90.
[299] Yeung MR, Loeng LL. Effects of joint attributes on tunnel stability. Int J Rock Mech Min Sci 1997;34(3/4), Paper No. 348.
[300] Hatzor YH, Benary R. The stability of a laminated Vousoir beam: back analysis of a historic roof collapse using DDA. Int J Rock Mech Min Sci 1998;35(2):165–81.
[301] Ohnishi Y, Chen G. Simulation of rock mass failure with discontinuous deformation analysis. J Soc Mater Sci Jpn 1999; 48(4):329–33.
[302] Pearce CJ, Thavalingam A, Liao Z, Bićanić N. Computational aspects of the discontinuous deformation analysis framework for modelling concrete fracture. Eng Fract Mech 2000;65:283–98.
[303] Hsiung SM, Shi G. Simulation of earthquake effects on underground excavations using Discontinuous Deformation Analysis (DDA). In: Elworth D, Tinucci JP, Heasley KA, editors. Rock mechanics in the national interest: Swets & Zeitlinger Lisse, 2001. p. 1413–20.
[304] Warburton PM. Application of a new computer model for reconstructing blocky rock geometry, analysing single rock stability and identifying keystones. Proceedings of the Fifth International Congress. Melbourne: ISRM, 1983. p. F225–F230.
[305] Warburton PM. Some modern developments in block theory for rock engineering. In: Hudson JA, editor In-chief. Comprehensive rock engineering, vol. 3, 1993. p. 293–315.
[306] Goodman RE, Shi G. Block theory and its application to rock engineering. Englewood Cliffs, NJ: Prentice-Hall, 1985.
[307] Shi G, Goodman RE. The key blocks of unrolled joint traces in developed maps of tunnel walls. Int J Numer Anal Methods Geomech 1989;13:131–58.
[308] Shi G, Goodman RE. Finding 3-d maximum key blocks on unrolled joint traces of tunnel surfaces. In: Hustrulid, Johnson, editors. Rock mechanics contributions and challenges. Rotterdam: Balkema, 1990. p. 219–28.
[309] Stone C, Young D. Probabilistic analysis of progressive block failures. In: Nelson PP, Laubach SE, editors. Rock mechanics: models and measurements, challenges from industry. Rotterdam: Balkema, 1994. p. 531–8.
[310] Mauldon M. Variation of joint orientation in block theory analysis. Int J Rock Mech Min Sci Geomech Abstr 1993; 30(7):1585–90.
[311] Hatzor YH. The block failure likelihood: a contribution to rock engineering in blocky rock masses. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1591–7.
[312] Jakubowski J, Tajdus A. The 3D Monte-Carlo simulation of rigid block around a tunnel. In: Rossmanith HP, editor. Mechanics of jointed and faulted rock. Rotterdam: Balkema, 1995. p. 551–6.
[313] Kuszmaul JS, Goodman RE. An analytical model for estimating key block sizes in excavations in jointed rock masses. In: Myer LR, Cook NGW, Goodman RE, Tsang CF, editors. Fractured and jointed rock masses. Rotterdam: Balkema, 1995. p. 19–26.

[314] Karaca M, Goodman RE. The influence of water on the behaviour of a key block. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1575–8.

[315] Mauldon M, Chou KC, Wu Y. Linear programming analysis of keyblock stability. In: Yuan J, editor. Computer methods and advancements in geomechanics, vol. 1. Rotterdam: Balkema, 1997. p. 517–22.

[316] Windsor CR. Block stability in jointed rock masses. In: Myer LR, Cook NGW, Goodman RE, Tsang CF, editors. Fractured and jointed rock masses. Rotterdam: Balkema, 1995. p. 59–66.

[317] Wibowo JL. Consideration of secondary blocks in key-block analysis. Int J Rock Mech Min Sci 1997;34(3/4), Paper No. 333.

[318] Chern JC, Wang MT. Computing 3-D key blocks delimited by joint traces on tunnel surfaces. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1599–604.

[319] Scott GA, Kottenstette JT. Tunnelling under the Apache Trial. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1485–9.

[320] Nishigaki Y, Miki S. Application of block theory in weathered rock. In: Myer LR, Cook NGW, Goodman RE, Tsang CF, editors. Fractured and jointed rock masses. Rotterdam: Balkema, 1995. p. 753–8.

[321] Yow JL. Block analysis for preliminary design of underground excavations. In: Hustrulid WA, Johnson GA, editors. Rock mechanics contributions and challenges. Rotterdam: Balkema, 1990. p. 429–35.

[322] Boyle WJ, Vogt TJ. Rock block analysis at Mount Rushmore National Memorial. In: Myer LR, Cook NGW, Goodman RE, Tsang CF, editors. Fractured and jointed rock masses. Rotterdam: Balkema, 1995. p. 717–23.

[323] Lee CI, Song JJ. Stability analysis of rock blocks around a tunnel. In: Rossmanith HP, editor. Mechanics of jointed and faulted rock. Rotterdam: Balkema, 1998. p. 443–8.

[324] Lee IM, Park JK. Stability analysis of tunnel keyblock: a case study. Tunnelling Underground Space Technol 2000;15(4): 453–62.

[325] Cundall PA, Strack ODL. A discrete numerical model for granular assemblies. Géotechnique 1979;29(1):47–65.

[326] Cundall PA, Strack ODL. The development of constitutive laws for soil using the distinct element method. In: Wittke W, editor. Numerical methods in geomechanics, Aachen, Germany, 1979. p. 289–98.

[327] Cundall PA, Strack ODL. The distinct element method as a tool for research in granular media. Report to NSF concerning grant ENG 76-20711, Part II, Department of Civil Engineering, University of Minnesota, 1979.

[328] Cundall PA, Strack ODL. Modelling of microscopic mechanisms in granular material. Proceedings of the US–Japan Sem. New Models Const. Rel. Mech. Gran. Mat., Itaca, 1982.

[329] Cundall PA. BALL—a program to model granular media using the distinct element method. London: Dames and Moore Advanced Technology Group, 1978.

[330] Strack ODL, Cundall PA. The distinct element method as a tool for research in granular media. Research Report, Part I. Department of Civil Engineering, University of Minnesota, 1978.

[331] Jenkins JT, Satake M, editors. Mechanics of granular materials: new models and constitutive relations. Amsterdam: Elsevier, 1983.

[332] Satake M, Jenkins JT, editors. Micromechanics of granular materials. Amsterdam: Elsevier, 1988.

[333] Biarez J, Gourvés R, editors. Proceedings of the International Conference Micromechanics of Granular Media, Clermont-Ferrand, 4–8 December 1989. Rotterdam: Balkema, 1989.

[334] Thornton C, editor. Proceedings of the Second International Conference Micromechanics of Granular Media, Birmingham, UK, 12–16 July 1993. Rotterdam: Balkema, 1983.

[335] Siriwardane HJ, Zaman MM, editors. Proceedings of the Eighth International Conference on Computer Methods and Advances in Geomechanics, West Virginia, USA, 22–28 May 1994. Rotterdam: Balkema, 1994.

[336] Mustoe GGW, Henriksen M, Huttelmaier HP, editors. Proceedings of the First Conference on DEM. CSM Press, Golden, CO, USA, 1989.

[337] Williams JR, Mustoe GGW, editors. Proceedings of the Second International Conference on DEM. IESL Publications, Boston, MIT, 1993.

[338] ITSACA Consulting Group, Ltd., PFC codes manuals, 1995.

[339] Taylor LM, Preece SD. DMC—a rigid body motion code for determining the interaction of multiple spherical particles. Research Report SND-88-3482. Sandia National Laboratory, USA, 1989.

[340] Taylor LM, Preece SD. Simulation of blasting induced rock motion using spherical element models. Eng Comput 1990;9: 243–52.

[341] Preece DS. Rock motion simulation of confined volume blasting. In: Hustrulis, Hihnson, editors. Rock mechanics contributions and challenges. Rotterdam: Balkema, 1990. p. 873–80.

[342] Preece DS. A numerical study of bench blast row delay timing and its influence on percent-cast. In: Siriwardane HJ, Zaman MM, editors. Computer methods and advances in geomechanics. Rotterdam: Balkema, 1994. p. 863–70.

[343] Preece DS, Knudsen SD. Computer modeling of gas flow and gas loading of rock in a bench blasting environment. In: Tillerson JR, Wawersik WR, editors. Rock mechanics. Rotterdam: Balkema, 1992. p. 295–303.

[344] Preece DS, Burchell SL, Scovira DS. Coupled explosive gas flow and rock motion modeling with comparison to bench blast field data. In: Rossmanith HP, editor. Rock fragmentation by blasting. Rotterdam: Balkema, 1993. p. 239–45.

[345] Preece DS, Scovira DS. Environmentally motivated tracking of geologic layer movement during bench blasting using discrete element method. In: Nelson PP, Laubach SE, editors. Rock mechanics. Rotterdam: Balkema, 1994. p. 615–22.

[346] Donzé FV, Bouchez J, Magnier SA. Modeling fractures in rock blasting. Int J Rock Mech Min Sci 1997;34(8):1153–63.

[347] Lee KW, Ryu CH, Synn JH, Park C. Rock fragmentation with plasma blasting method. In: Lee KH, Yang HS, Chung SK, editors. Environmental and safety concerns in underground construction. Rotterdam: Balkema, 1997. p. 147–52.

[348] Lin X, Ng TT. Numerical modeling of granular soil using random arrays of three-dimensional elastic ellipsoids. In: Siriwardane HJ, Zaman MM, editors. Proceedings of the Eighth International Conference on Computer Methods and Advances in Geomechanics. Rotterdam: Balkema, 1994. p. 605–10.

[349] Iwashita K, Tarumi Y, Casaverde L, Uemura D, Meguro K, Hakuno M. Granular assembly simulation for ground collapse. In: Satake M, Jenkins JT, editors. Micromechanics of granular materials. Amsterdam: Elsevier, 1988. p. 125–32.

[350] Zhai ED, Miyajima M, Kitaura M. DEM simulation of rise of excess pore water pressure of saturated sands under vertical ground motion. In: Yuan J, editor. Computer methods and advances in geomechanics, vol. 1. Rotterdam: Balkema, 1997. p. 535–9.

[351] Thallak S, Rothenburg L, Dusseault M. Simulation of multiple hydraulic fracture in a discrete element system. In: Rogiers JC, editor. Rock mechanics as a multidisciplinary science. Rotterdam: Balkema, 1991. p. 271–80.

[352] Huang JI, Kim K. Fracture process zone development during hydraulic fracturing. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1295–8.

[353] Kim K, Yao C. The influence of constitutive behaviour on the fracture process zone and stress field evolution during hydraulic fracturing. In: Nelson PP, Laubach SE, editors. Rock mechanics: models and measurements—challenges from industry. Rotterdam: Balkema, 1994. p. 193–200.
[354] Kiyama H, Fujimura H, Nishimura T, Tanimoto C. Distnct element analysis of the Fenner–Pacher type characteristic vurve for tunneling. In: Wittke W, editor. Proceedings of the Seventh Congress of ISRM, Aachen, Germany, vol. 1, 1991. p. 769–72.
[355] Blair SC, Cook NGW. Statistical model of rock fracture. In: Tillerson JR, Wawersik WR, editors. Rock Mechanics. Rotterdam: Balkema, 1992. p. 729–35.
[356] Paterson L. Serrated fracture growth with branching. In: Cundall PA, Sterling R, Starfield AM, editors. Key questions in rock mechanics. Rotterdam: Balkema, 1988. p. 351–8.
[357] Song J, Kim K. Dynamic modelling of stress wave induced fracture. In: Siriwardane HJ, Zaman MM, editors. Computer methods and advances in geomechanics. Rotterdam: Balkema, 1994. p. 877–81.
[358] Song J, Kim K. Numerical simulation of delay blasting by the dynamic lattice network model. In: Nelson PP, Laubach SE, editors. Rock mechanics. Rotterdam: Balkema, 1994. p. 301–7.
[359] Song J, Kim K. Numerical simulation of the blasting induced disturbed rock zone using the dynamic lattice network model. In: Rossmanith HP, editor. Mechanics of jointed and faulted rock. Rotterdam: Balkema, 1995. p. 755–61.
[360] Mühlhaus HB, Sakaguchi H, Wei Y. Particle based modelling of dynamic fracture in jointed rock. In: Yuan J, editor. Computer methods and advances in geomechanics, vol. 1. Rotterdam: Balkema, 1997. p. 207–16.
[361] Schlangen E, van Mier JGM. Fracture simulation in concrete and rock using a random lattice. In: Siriwardane HJ, Zaman MM, editors. Computer methods and advances in geomechanics. Rotterdam: Balkema, 1994. p. 1641–6.
[362] Li LH, Bai YL, Xia MF, Ke FJ, Yin XC. Damage localization as a possible precursor of earthquake rupture. Pure Appl Geophys 2000;157:1929–43.
[363] Napier JAL, Dede T. A comparison between random mesh schemes and explicit growth rules for rock fracture simulation. Int J Rock Mech Min Sci 1997;34(3/4):356 (Paper No. 217).
[364] Place D, Mora P. Numerical simulation of localization phenomena in a fault zone. Pure Appl Geophys 2000;157: 1821–45.
[365] Long JCS, Remer JS, Wilson CR, Witherspoon PA. Porous media equivalents for networks of discontinuous fractures. Water Resour Res 1982;18(3):645–58.
[366] Long JCS, Gilmour P, Witherspoon PA. A model for steady fluid flow in random three dimensional networks of disc-shaped fractures. Water Resour Res 1985;21(8):1105–15.
[367] Robinson PC. Connectivity, flow and transport in network models of fractured media. PhD thesis, St. Catherine's College, Oxford University, UK, 1984.
[368] Andersson J. A stochastic model of a fractured rock conditioned by measured information. Water Resour Res 1984;20(1):79–88.
[369] Endo HK. Mechanical transport in two-dimensional networks of fractures. PhD thesis, University of California, Berkeley, 1984.
[370] Endo HK, Long JCS, Wilson CK, Witherspoon PA. A model for investigating mechanical transport in fractured media. Water Resour Res 1984;20(10):1390–400.
[371] Smith L, Schwartz FW. An analysis of the influence of fracture geometry on mass transport in fractured media. Water Resour Res 1984;20(9):1241–52.
[372] Elsworth D. A model to evaluate the transient hydraulic response of three-dimensional sparsely fractured rock masses. Water Resour Res 1986;22(13):1809–19.
[373] Elsworth D. A hybrid boundary-element-finite element analysis procedure for fluid flow simulation in fractured rock masses. Int J Numer Anal Methods Geomech 1986;10:569–84.
[374] Dershowitz WS, Einstein HH. Three-dimensional flow modelling in jointed rock masses. In: Herget O, Vongpaisal O, editors. Proceedings of the Sixth Congress on ISRM, Montreal, Canada, vol, 1, 1987. p. 87–92.
[375] Andersson J, Dverstop B. Conditional simulations of fluid flow in three-dimensional networks of discrete fractures. Water Resour Res 1987;23(10):1876–86.
[376] Yu Q, Tanaka M, Ohnishi Y. An inverse method for the model of water flow in discrete fracture network. Proceedings of the 34th Janan National Conference on Geotechnical Engineering, Tokyo, 1999. p. 1303–4.
[377] Zimmerman RW, Bodvarsson GS. Effective transmissivity of two-dimensional fracture networks. Int J Rock Mech Min Sci Geomech Abstr 1996;33(4):433–6.
[378] Bear J, Tsang CF, de Marsily G. Flow and contaminant transport in fractured rock. San Diego: Academic Press, 1993.
[379] Sahimi M. Flow and transport in porous media and fractured rock. Weinheim: VCH Verlagsgesellschaft mbH, 1995.
[380] National Research Council. Rock fractures and fluid flow—contemporary understanding and applications. Washington, DC: National Academy Press, 1996.
[381] Adler PM, Thovert JF. Fractures and fracture networks. Dordrecht: Kluwer Academic Publishers, 1999.
[382] Dershowitz WS, Lee G, Geier J, Hitchcock S, la Pointe P. User documentation: Fracman discrete feature data analysis, geometric modelling and exploration simulations. Seattle: Golder Associates, 1993.
[383] Stratford RG, Herbert AW, Jackson CP. A parameter study of the influence of aperture variation on fracture flow and the consequences in a fracture network. In: Barton N, Stephansson O, editors. Rock joints. Rotterdam: Balkema, 1990. p. 413–22.
[384] Herbert AW. NAPSAC (Release 3.0) summary document. AEA D&R 0271 Release 3.0, AEA Technology, Harwell, UK, 1994.
[385] Herbert AW. Modelling approaches for discrete fracture network flow analysis. In: Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media-mathematical and experimental studies. Amsterdam: Elsevier, 1996. p. 213–29.
[386] Wilcock P. The NAPSAC fracture network code. In: Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media. Rotterdam: Elsevier, 1996. p. 529–38.
[387] Dershowitz WS. Rock joint systems. PhD thesis, Massachusetts Institute of Technology, Boston, USA, 1984.
[388] Billaux D, Chiles JP, Hestir K, Long JCS. Three-dimensional statistical modeling of a fractured rock mass—an example from the Fanay-Augéres Mine. Int J Rock Mech Min Sci Geomech Abstr 1989;26(3/4):281–99.
[389] Mauldon M. Estimating mean fracture trace length and density from observations in convex windows. Rock Mech Rock Eng 1998;31(4):201–16.
[390] Mauldon M, Dunne WM, Rohrbaugh Jr MB. Circular scanlines and circular windows: new tools for characterizing the geometry of fracture traces. J Struct Geol 2001;23:247–58.
[391] Long JCS. Investigation of equivalent porous media permeability in networks of discontinuous fractures. PhD thesis, Lawrence Berkeley Laboratory, University of California, Berkeley, CA, 1983.
[392] Amadei B, Illangasekare T. Analytical solutions for steady and transient flow in non-homogeneous and anisotropic rock joints. Int J Rock Mech Min Sci Geomech Abstr 1992; 29(6):561–72.

[393] Robinson PC. Flow modelling in three dimensional fracture networks. Research Report, AERE-R-11965, UK AEA, Harwell, 1986.
[394] Cacas MC, Ledoux E, de Marsily G, Tille B, Barbreau A, Durand E, Feuga B, Peaudecerf P. Modeling fracture flow with a stochastic discrete fracture network: calibration and validation. I. The flow model. Water Resour Res 1990;26(3):479–89.
[395] Tsang YW, Tsang CF. Channel model of flow through fractured media. Water Resour Res 1987;22(3):467–79.
[396] Barton CC, Larsen E. Fractal geometry of two-dimensional fracture networks at Yucca Mountain, southwestern Nevada. In: Stephansson O, editor. Proceedings of the International Symposium on Rock Joints, Bjorkliden, Sweden, 1985. p. 77–84.
[397] Chilés JP. Fractal and geostatistical methods for modelling a fracture network. Math Geol 1988;20(6):631–54.
[398] Barton CC. Fractal analysis of the scaling and spatial clustering of fractures in rock. In: Fractals and Their Application to Geology. Proceedings of the 1988 GSA Annual Meeting, 1992.
[399] Renshaw CE. Connectivity of joint networks with power law length distribution. Water Resour Res 1999;35(9):2661–70.
[400] Sudicky EA, McLaren RG. The Laplace transform Galerkin technique for large scale simulation of mass transport in discretely fractured porous formations. Water Resour Res 1992; 28(2):499–514.
[401] Dershowitz W, Miller I. Dual porosity fracture flow and transport. Geophys Res Lett 1995;22(11):1441–4.
[402] Pruess K, Wang JSY. Numerical modelling of isothermal and non-isothermal flow in unsaturated fractured rock—a review. In: Evan, Nicholson, editors. Flow and transport through unsaturated fractured rocks, Monograph 42. Washington, DC: American Geophysical Union, 1987. p. 11–21.
[403] Slough KJ, Sudicky EA, Forsyth PA. Numerical simulation of multiphase flow and phase partitioning in discretely fractured geologic media. J Contaminant Hydrol 1999;40:107–36.
[404] Hughes RG, Blunt MJ. Network modelling of multiphase flow in fractures. Adv Water Res 2001;24:409–21.
[405] Layton GW, Kingdon RD, Herbert AW. The application of a three-dimensional fracture network model to a hot-dry-rock reservoir. In: Tillerson JR, Wawersik WR, editors. Rock mechanics. Rotterdam: Balkema, 1992. p. 561–70.
[406] Ezzedine S, de Marsily G. Study of transient flow in hard fractured rocks with a discrete fracture network model. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1605–9.
[407] Watanabe K, Takahashi H. Parametric study of the energy extraction from hot dry rock based on fractal fracture network model. Geothermics 1995;24(2):223–36.
[408] Kolditz O. Modelling flow and heat transfer in fracture rocks: conceptual model of a 3-d deterministic fracture network. Geothermics 1995;24(3):451–70.
[409] Willis-Richards J, Wallroth T. Approaches to the modeling of HDR reservoirs: a review. Geothermics 1995;24(3):307–32.
[410] Willis-Richards J. Assessment of HDR reservoir simulation and performance using simple stochastic models. Geothermics 1995;24(3):385–402.
[411] Dershowitz WS, Wallmann P, Doe TW. Discrete feature dual porosity analysis of fractured rock masses: applications to fractured reservoirs and hazardous waste. In: Tillerson, Wawersik, editors. Rock mechanics. Rotterdam: Balkema, 1992. p. 543–50.
[412] Herbert AW, Layton GW. Discrete fracture network modeling of flow and transport within a fracture zone at Stripa. In: Myer LR, Cook NGW, Goodman RE, Tsang CF, editors. Fractured and jointed rock masses. Rotterdam: Balkema, 1995. p. 603–9.
[413] Doe TW, Wallmann PC. Hydraulic characterization of fracture geometry for discrete fracture modelling. Proceedings of the Eighth Congress on IRAM, Tokyo, 1995. p. 767–72.
[414] Barthélémy P, Jacquin C, Yao J, Thovert JF, Adler PM. Hierarchical structures and hydraulic properties of a fracture network in the Causse of Larzac. J Hydrol 1996;187: 237–58.
[415] Jing L, Stephansson O. Network topology and homogenization of fractured rocks. In: Jamtveit B, editor. Fluid flow and transport in rocks: mechanisms and effects. London: Chapman & Hall, 1996. p. 91–202.
[416] Margolin G, Berkowitz B, Scher H. Structure, flow and generalized conductivity scaling in fractured networks. Water Resour Res 1998;34(9):2103–21.
[417] Mazurek M, Lanyon GW, Vomvoris S, Gautschi A. Derivation and application of a geologic dataset for flow modeling by discrete fracture networks in low-permeability argillaceous rocks. J Contaminant Hydrol 1998;35:1–17.
[418] Zhang X, Sanderson DJ. Scale up of two-dimensional conductivity tensor for heterogenous fracture networks. Eng Geol 1999;53:83–99.
[419] Rouleau A, Gale JE. Stochastic discrete fracture simulation of groundwater flow into underground excavation in granite. Int J Rock Mech Min Sci Geomech Abstr 1987;24(2): 99–112.
[420] Xu J, Cojean R. A numerical model for fluid flow in the block interface network of three dimensional rock block system. In: Rossmanith HP, editor. Mechanics of jointed and faulted rock. Rotterdam: Balkema, 1990. p. 627–33.
[421] He S. Research on a model of seapage flow of fracture networks and modelling for coupled hydro-mechanical processes in fractured rock masses. In: Yuan J, editor. Computer methods and advances in geomechanics, vol. 2. Rotterdam: Balkema, 1997. p. 1137–42.
[422] Hestir K, Long JCS. Analytical expressions for the permeability of random two-dimensional networks based on regular lattice percolation and equivalent media theories. J Geophys Res 1990; 95(B13):21565–81.
[423] Berkowitz B, Balberg I. Percolation theory and its applications to groundwater hydrology. Water Resour Res 1993;29(4): 775–94.
[424] Zhang X, Sanderson DJ. Numerical study of critical behaviour of deformation and permeability of fractured rocks. Mar Pet Geol 1998;15:535–48.
[425] Mo H, Bai M, Lin D, Roegiers JC. Study of flow and transport in fracture networks using percolation theory. Appl Math Modelling 1998;22:277–91.
[426] Guéquen Y, Chelidze T, Le Ravalec M. Microstructures, percolation thresholds, and rock physical properties. Tectonophysics 1997;279:23–35.
[427] Kimich R, Klemm A, Weber M. Flow, diffusion, and thermal convection in percolation clusters: NMR experiments and numerical FEM/FVM simulations. Magn Reson Imaging 2001; 19:353–61.
[428] Zienkiewicz OC, Kelly DW, Bettess P. The coupling of the finite element method and boundary solution procedures. Int J Numer Methods Eng 1977;11:355–75.
[429] Brady BHG, Wassyng A. A coupled finite element—boundary element method of stress analysis. Int J Rock Mech Min Sci Geomech Abstr 1981;18:475–85.
[430] Beer G. Finite element, boundary element and coupled analysis of unbounded problems in elastostatics. Int J Numer Methods Eng 1983;19:567–80.
[431] Varadarajan A, Sharma KG, Singh RB. Some aspects of coupled FEBEM analysis of underground openings. Int J Numer Anal Methods Geomech 1985;9:557–71.
[432] Ohkami T, Mitsui Y, Kusama T. Coupled boundary element/finite element analysis in geomechanics including body forces. Comput Geotech 1985;1:263–78.

[433] Gioda G, Carini A. A combined boundary element–finite element analysis of lined openings. Rock Mech Rock Eng 1985;18:293–302.
[434] Swoboda G, Mertz W, Beer G. Pheological analysis of tunnel excavations by means of coupled finite element (FEM)–boundary element (BEM) analysis. Int J Numer Anal Methods Geomech 1987;11:15–129.
[435] von Estorff O, Firuziaan M. Coupled BEM/FEM approach for non-linear soil/structure interaction. Eng Anal Boundary Elements 2000;24:715–25.
[436] Lorig LJ, Brady BHG. A hybrid discrete element–boundary element method of stress analysis. In: Goodman RE, Heuze F, editors. Proceedings of the 23rd US Symposium Rock Mechanics, Berkeley, 25-27 August 1982. p. 628–36.
[437] Lorig LJ, Brady BHG, Cundall PA. Hybrid distinct element–boundary element analysis of jointed rock. Int J Rock Mech Min Sci Geomech Abstr 1986;23(4):303–12.
[438] Wei L. Numerical studies of the hydromechanical behaviour of jointed rocks. PhD thesis, Imperial College of Science and Technology, University of London, 1992.
[439] Wei L, Hudson JA. A hybrid discrete–continuum approach to model hydro-mechanical behaviour of jointed rocks. Eng Geol 1988;49:317–25.
[440] Pan XD, Reed MB. A coupled distinct element–finite element method for large deformation analysis of rock masses. Int J Rock Mech Min Sci Geomech Abstr 1991;28(1):93–9.
[441] Pöttler R, Swoboda GA. Coupled beam–boundary element model (FE-BEM) for analysis of underground openings. Comput Geotech 1986;2:239–56.
[442] Sugawara K, Aoki T, Suzuki Y. A coupled boundary element-characteristics method for elasto-plastic analysis of rock caverns. In: Romana, editor. Rock mechanics and power plants. Rotterdam: Balkema, 1988. p. 248–58.
[443] Millar D, Clarici E. Investigation of back-propagation artificial neural networks in modelling the stress–strain behaviour of sandstone rock. IEEE International Conference on Neural Networks—Conference Proceedings, vol. 5, 1994. p. 3326–31.
[444] Alvarez Grima M, Babuka R. Fuzzy model for the prediction of unconfined compressive strength of rock samples. Int J Rock Mech Min Sci 1999;36(3):339–49.
[445] Singh VK, Singh D, Singh TN. Prediction of strength properties of some schistose rocks from petrographic properties using artificial neural networks. Int J Rock Mech Min Sci 2001; 38(2):269–84.
[446] Kacewicz M. Model-free estimation of fracture apertures with neural networks. Math Geol 1994;26(8):985–94.
[447] Lessard JS, Hadjigeorgiou J. Modelling shear behavior of rough joints using neural networks. In: Vouille G, Berest P, editors. 20th Century Lessons, 21st Century Challenges, ISBN 9058090698, 1999. p. 925–9.
[448] Sirat M, Talbot CJ. Application of artificial neural networks to fracture analysis at the Äspö HRL, Sweden: fracture sets classification. Int J Rock Mech Min Sci 2001;38(5):621–39.
[449] Qiao C, Zhang Q, Huang X. Neural network method for evaluation of mechanical parameters of rock mass in numerical simulation. Yanshilixue Yu Gongcheng Xuebao/Chin J Rock Mech Eng 2000;19(1):64–7.
[450] Feng XT, Zhang Z, Sheng Q. Estimating mechanical rock mass parameters relating to the Three Gorges Project permanent shiplock using an intelligent displacement back analysis method. Int J Rock Mech Min Sci 2000;37(7):1039–54.
[451] Feng XT, Seto M. A new method of modeling the rock microfracturing problems in double torsion experiments using neural networks. Int J Numer Anal Meth 1999;23(9):905–23.
[452] Sklavounos P, Sakellariou M. Intelligent classification of rock masses. Appl Artif Intell Eng 1995;387–93.
[453] Liu Y, Wang J. A neural network method for engineering rock mass classification. In: Vouille G, Berest P, editors. 20th Century Lessons, 21st Century Challenges. ISBN 9058090698, 1999. p. 519–22.
[454] Deng JH, Lee CF. Displacement back analysis for a steep slope at the Three Gorges Project site. Int J Rock Mech Min Sci 2001;38(2):259–68.
[455] Alvarez Grima M, Bruines PA, Verhoef PNW. Modelling tunnel boring machine performance by neuro-fuzzy methods. Tunnelling Underground Space Technol 2000;15(3):259–69.
[456] Sellner PJ, Steindorfer AF. Prediction of displacements in tunnelling. Felsbau 2000;18(2):22–6.
[457] Lee C, Sterling R. Identifying probable failure modes for underground openings using a neural network. Int J Rock Mech Min Sci 1992;29(1):49–67.
[458] Leu SS. Data mining for tunnel support stability: neural network approach. Autom Constr 2001;10(4):429–41.
[459] Leu SS, Chen CN, Chang SL. Data mining for tunnel support stability: neural network approach. Autom Constr 2001;10(4): 429–41.
[460] Kim CY, Bae GJ, Hong SW, Park CH, Moon HK, Shin HS. Neural network based prediction of ground surface settlements due to tunnelling. Comput Geotech 2001;28(6–7):517–47.
[461] Feng XT, Seto M, Katsuyama K. Neural network modelling on earthquake magnitude series. Int J Geophys 1997;128(3): 547–56.
[462] Millar DL, Hudson JA. Performance monitoring of rock engineering systems using neural networks. Trans Inst Min Metall A 1994;103:A3–16.
[463] Yang Y, Zhang Q. Application of neural networks to Rock Engineering Systems (RES). Int J Rock Mech Min Sci 1998; 35(6):727–45.
[464] Yi H, Wanstedt S. The introduction of neural network system and its applications in rock engineering. Eng Geol 1998;49 (3–4):253–60.
[465] Hudson JA, Hudson JL. Rock mechanics and the Internet. Int J Rock Mech Min Sci 1997;34(3–4):603 (with full paper on the associated CD).
[466] Feng XT, Hudson JA. The ways ahead for rock engineering design methodologies, 2003 (manuscript in preparation).
[467] Singh B. Continuum characterization of jointed rock masses. Part 1—the constitutive equations. Int J Rock Mech Min Sci Geomech Abstr 1973;22(4):197–213.
[468] Gerrard CM. Equivalent elastic moduli of a rock mass consisting of orthorhombic layers. Int J Rock Mech Min Sci Geomech Abstr 1982;19:9–14.
[469] Fossum AF. Effective elastic properties for a random jointed rock mass. Int J Rock Mech Min Sci Geomech Abstr 1985;22(6):467–70.
[470] Wei ZQ, Hudson JA. The influence of joints on rock modulus. Proceedings of the International Symposium Rock Engineering in Complex Rock Formations. Beijing, China: Science Press, 1986. p. 54–62.
[471] Yashinaka R, Yamabe T. Joint stiffness and the deformation behaviour of discontinuous rock. Int J Rock Mech Min Sci Geomech Abstr 1986;23(1):19–28.
[472] Wu FQ. A 3D model of a jointed rock mass and its deformation properties. Int J Min Geol Eng 1988;6:169–76.
[473] Mukarami H, Hegemier GA. Development of a non-linear continuum model for wave propagation in jointed media: theory for single joint set. Mech Mater 1989;8:199–218.
[474] Chen EP. A constitutive model for jointed rock mass with orthogonal sets of joints. J Appl Mech Trans ASME 1989;56: 25–32.
[475] Singh M. Applicability of a constitutive model to jointed block mass. Rock Mech Rock Eng 2000;33(2):141–7.

[476] Wu FQ, Wang SJ. A stress–strain relation for jointed rock masses. Int J Rock Mech Min Sci 2001;38:591–8.

[477] Oda M. An equivalent continuum model for coupled stress and fluid flow analysis in jointed rock masses. Water Resour Res 1986;22(13):1845–56.

[478] Papamichos E. Constitutive laws for geomaterials. Oil Gas Sci Technol—Rev IFP 1999;54(6):759–71.

[479] Hoek E. Strength of jointed rock masses. 1983 Rankine Lecture. Géotechnique 1983;33(3):187–223.

[480] Hoek E, Brown ET. Practical estimates of rock mass strength. Int J Rock Mech Min Sci 1997;34(8):1165–86.

[481] Zheng Y, Chu J, Xu Z. Strain space formulation of the elasto-plastic theory and its finite element implementation. Comput Geotech 1986;2:373–88.

[482] Adhikary DP, Dyskin AV. A continuum model of layered rock masses with non-associative joint plasticity. Int J Numer Anal Methods Geomech 1998;22:245–61.

[483] Sulem J, Vardoulakis I, Papamichos E, Oulahna A, Tronvoll J. Elasto-plastic modelling of Red Wildmoor sandstone. Mech Cohes-Frict Mater 1999;4:215–45.

[484] Boulon M, Alachaher A. A new incrementally non-linear constitutive law for finite element applications in geomechanics. Comput Struct 1995;17:177–201.

[485] Dragon A, Mróz Z. A continuum model for plastic-brittle behaviour of rock and concrete. Int J Eng Sci 1979;17:121–37.

[486] Nemat-Nasser S. On finite plastic flow of crystalline solids and geomaterials. J Appl Mech Trans ASME 1983;50:1114–26.

[487] Zienkiewicz OC, Mróz Z. Generalized plasticity formulation and applications to geomechanics. In: Desai CS, Gallagher RH, editors. Mechanics of engineering materials. New York: Wiley, 1984. p. 655–79.

[488] Read HE, Hegemier GA. Strain softening of rock, soil and concrete—review article. Mech Mater 1984;3:194–271.

[489] Gerrard CM, Pande GN. Numerical modelling of reinforced jointed rock masses. Comput Geotech 1985;1:293–318.

[490] Desai CS, Salami MR. Constitutive model for rocks. J Geotech Eng Trans ASCE 1987;113(5):407–23.

[491] Rowshandel B, Nemat-Nasser S. Finite strain rock plasticity: stress triaxiality, pressure, and temperature effects. J Soil Dyn Earthquake Eng 1987;6(4):203–19.

[492] Kim MK, Lade PV. Single hardening constitutive model for frictional materials. Comput Geotech 1988;5:307–24.

[493] Sterpi D. An analysis of geotechnical problems involving strain softening effects. Int Numer Anal Meth Geomech 1999;23:1427–54.

[494] Rudnicki JW, Rice JR. Conditions for the localization of deformation in pressure-sensitive dilatant materials. J Mech Phys Solids 1975;23:371–94.

[495] Pariseau WG. On the significance of dimensionless failure criteria. Int J Rock Mech Min Sci Geomech Abstr 1994;31(5):555–60.

[496] Sheorey PR. Empirical rock failure criteria. Rotterdam: Balkema, 1997.

[497] Mostyn G, Douglas K. Strength of intact rock and rock masses. Proceedings of the Geological Engineering 2000. International Conference on Geotechnical and Geological Engineering, 19–24 November 2000, Melbourne, Australia, vol. 1. Lancaster: Technomic Publishing Co., Inc., 2000. p. 1389–421.

[498] Parry RHG. Shear strength of geomaterials—a brief historical perspective. Proceedings of the Geological Engineering 2000 (International Conference on Geotechnical and Geological Engineering, 19–24 November 2000, Melbourne, Australia), vol. 1. Lancaster: Technomic Publishing Co., Inc., 2000. p. 1592–617.

[499] Amadei B, Savage WZ. Anisotropic nature of jointed rock mass strength. J Eng Mech Trans ASCE 1989;115(3):525–42.

[500] Sun J, Hu YY. Time-dependent effects on the tensile strength of saturated granite at Three-Gorges Project in China. Int J Rock Mech Min Sci 1997;34(3/4):381 (Paper No. 361).

[501] Duvean G, Shao JF, Henry JP. Assessment of some failure criteria for strongly anisotropic geomaterials. Mech Cohes-Frict Mater 1998;3:1–26.

[502] Gupta V, Bergström JS. Compressive failure of rocks. Int J Rock Mech Min Sci 1997;34(3/4):376 (Paper No. 112).

[503] Tharp TP. Time-dependent compressive failure around an opening. Int J Rock Mech Min Sci 1997;34(3/4):380 (Paper No. 310).

[504] Hawkins AB. Aspects of rock strength. Bull Eng Geol Environ 1998;57:17–30.

[505] Kumar P. Shear failure envelope of Hoek–Brown criterion for rockmass. Tunnelling Underground Space Technol 1998;13(4):453–8.

[506] Singh B, Goel RK, Mehrotra VK, Garg SK, Allu MR. Effect of intermediate principal stress on strength of anisotropic rock masses. Tunnelling Underground Space Technol 1998;13(1):71–9.

[507] Lai YS, Wang CY, Tien YM. Modified Mohr–Coulomb-type micromechanical failure criteria for layered rocks. Int J Numer Anal Methods Geomech 1999;23:451–60.

[508] Aubertin M, Li L, Simon R. A multiaxial stress criterion for short- and long-term strength of isotropic rock media. Int J Rock Mech Min Sci 2000;37:1169–93.

[509] Hibino S, Motojima M, Hayashi M. Proposed failure criterion and non-linear deformability relationship for rock and granular materials. Proceedings of the Geological Engineering 2000 (International Conference on Geotechnical and geological Engineering, 19–24 November 2000, Melbourne, Australia), vol. 2. Lancaster: Technomic Publishing Co., Inc., 2000. p. 514.

[510] Masuda K. Effects of water on rock strength in a brittle regime. J Struct Geol 2001;23:1653–7.

[511] Tien YM, Kuo MC. A failure criterion for transversely isotropic rocks. Int J Rock Mech Min Sci 2001;38:399–412.

[512] Jaeger JC, Cook NGW. Fundamentals of rock mechanics. London, UK: Methuen, 1969.

[513] Cristescu ND, Hunsche U. The time effects in rock mechanics. Chichester, UK: Wiley, 1998.

[514] Valanis KC, editor. Constitutive equations in viscoplasticity: phenomenological and physical aspects. Proceedings of the Winter Annual Meeting of the ASME, New York, 5–10 December 1976. New York: ASME, 1976.

[515] Ohkami T, Ichikawa Y. A parameter identification procedure for viscoelastic materials. Comput Geotech 1997;21(4):255–75.

[516] Ohkami T, Swoboda G. Parameter identification of viscoelastic materials. Comput Geotech 1999;24:279–95.

[517] Yang Z, Wang Z, Zhang L, Zhou R, Xing N. Back-analysis of viscoelastic displacements in a soft rock road tunnel. Int J Rock Mech Min Sci 2001;38:331–41.

[518] Nawrocki PA, Mróz Z. A constitutive model for rock accounting for viscosity and yield stress degradation. Comput Geotech 1999;25:247–80.

[519] Nedjar B. Frameworks for finite strain viscoelastic-plasticity based on multiplicative decompositions. Part I: continuum formulations. Comput Methods Appl Mech Eng 2002;19:1541–62.

[520] Nedjar B. Frameworks for finite strain viscoelastic-plasticity based on multiplicative decompositions. Part II: computational aspects. Comput Methods Appl Mech Eng 2002;19:1563–93.

[521] Ronesson K, Ristinmaa M, Mähler L. A comparison of viscoplasticity formats and algorithms. Mech Cohes-Frict Mater 1999;4:75–98.

[522] Abousleiman Y, Cheng AHD, Jiang C, Roegiers JC. A micromechanically consistent poroviscoelasticity theory for rock mechanics application. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):1177–80.
[523] Abousleiman Y, Cheng AHD, Jiang C, Roegiers JC. Poroviscoelastic analysis of borehole and cylinder problems. Acta Mech 1996;119:199–219.
[524] Diez P, Arroyo M, Huerta A. Adaptivity based on error estimation for viscoplastic softening materials. Mech Cohes-Frict Mater 2000;5:87–112.
[525] Patton TL, Fletcher RC. A rheological model for fractured rock. J Struct Geol 1998;20(5):491–502.
[526] Shao JF, Duveau G, Hoteit N, Sibat M, Bart M. Time dependent continuous damage model for deformation and failure of brittle rock. Int J Rock Mech Min Sci 1997;34(3/4):385 (Paper No. 285).
[527] Fardin N, Jing L, Stephansson O. The scale dependence of rock joint surface roughness. Int J Rock Mech Min Sci 2001;38:659–69.
[528] Bažant Z. Size effect. Int J Solids Struct 2000;37:69–80.
[529] Aifantis EC. On the role of gradients in the localization of deformation and fracture. Int J Eng Sci 1992;30:1279–99.
[530] Zbib HM, Aifantis EC. A gradient-dependent flow theory of plasticity. Appl Mech Rev 1989;42:295–305.
[531] Zbib HM, Aifantis EC. On the gradient-dependent theory of plasticity and shear-banding. Acta Mech 1992;92:209–25.
[532] Frantziskonis G, Konstantinidis A, Aifantis EC. Scale-dependent constitutive relations and the role of scale on nominal properties. Eur J Mech A/Solids 2001;20:925–36.
[533] Fraldi M, Guarracino F. On a general property of a class of homogenized porous media. Mech Res Commun 2001;28(2):213–21.
[534] Snow DT. A parallel plate model of fractured permeable media. PhD thesis, University of California, Berkeley, 1965.
[535] Renard Ph, de Marsily G. Calculating equivalent permeability: a review. Adv Water Res 1997;20(5–6):253–78.
[536] Wen X, Gómez-Hernández J. Upscaling hydraulic conductivity in heterogeneous media: an overview. J Hydrol 1996;183:ix–xxii.
[537] Lee CH, Deng BW, Chang JL. A continuum approach for estimating permeability in naturally fractured rocks. Eng Geol 1995;39:71–85.
[538] Li D, Cullik AS, Lake LW. Global scale-up of reservoir model permeability with local grid refinement. J Pet Sci Eng 1995;14:1–13.
[539] Tran T. The 'missing scale' and direct simulation of block effective properties. J Hydrol 1996;183:37–56.
[540] Hristopulos DT, Christakos G. An analysis of hydraulic conductivity upscaling. Nonlinear Anal: Theory Method Appl 1997;30(8):4979–84.
[541] Scheibe T, Yabusaki S. Scaling of flow and transport behaviour in heterogeneous groundwater systems. Adv Water Res 1998;22(3):223–38.
[542] Pozdniakov SP, Tsang CF. A semianalytical approach to spatial averaging of hydraulic conductivity in heterogeneous aquifers. J Hydrol 1999;216:78–98.
[543] Shahimi M, Mehrabi AR. Percolation and flow in geological formations: upscaling from microscopic to megascopic scales. Physica A 1999;266:136–52.
[544] Lunati I, Bernard D, Giudici M, Parravicini G, Ponzini G. A numerical comparison between two upscaling techniques: non-local inverse based scaling and simplified renormalization. Adv Water Res 2001;24:913–29.
[545] Kachanov M, Sevostianov I, Shafiro B. Explicit cross-property correlations for porous materials with anisotropic microstructure. J Mech Phys Solids 2001;49:1–25.
[546] Quintard M, Kaviany M, Whitaker S. Two-medium treatment of heat transfer in porous media: numerical results for effective properties. Adv Water Res 1997;20(2–3):77–94.
[547] Zimmerman RW, Hadgu T, Bodvarsson GS. A new lumped-parameter model for flow in unsaturated dual-porosity media. Adv Water Res 1996;19(5):317–27.
[548] Bai M. An efficient algorithm for evaluating coupled processes in radial fluid flow. Comput Geotech 1997;23(2):195–202.
[549] Bai M, Meng F, Elsworth D, Abousleiman Y, Roegiers JC. Numerical modeling of coupled flow and deformation in fractured rock specimens. Int J Numer Anal Methods Geomech 1999;23:141–60.
[550] Choi ES, Cheema T, Islam MR. A new dual-porosity/dual-permeability model with non-Darcian flow through fractures. J Pet Sci Eng 1997;17:331–44.
[551] Masters I, Pao WKS, Lewis RW. Coupling temperature to a double-porosity model of deformable porous media. Int J Numer Anal Methods Geomech 2000;49:421–38.
[552] McLaren R, Forsyth PA, Sudicky EA, Vanderkwaak JE, Schwartz FW, Kessler JK. Flow and transport in fractured tuff at Yucca Mountain: numerical experiments on fast preferential flow mechanisms. J Contaminant Hydrol 2000;43:211–38.
[553] Vogel T, Gerke HH, Zhang R, Van Genuchten MT. Modeling flow and transport in a two-dimensional dual-permeability system with spatial variable hydraulic properties. J Hydrol 2000;238:78–89.
[554] Zhang K, Woodbury AD, Dunbar WS. Application of the Lancros algorithm to the simulation of groundwater flow in dual-porosity media. Adv Water Res 2000;23:579–89.
[555] Landereau P, Noetinger B, Quintard M. Quasi-steady two-equation models for diffusive transport in fractured porous media: large-scale properties for densely fractured systems. Adv Water Res 2001;24:863–76.
[556] Stietel A, Millard A, Treille E, Vuillod E, Thoraval A, Ababou A. Continuum representation of coupled hydromechanical processes of fractured media: homogenisation and parameter identification. In: Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media. Amsterdam: Elsevier, 1996. p. 135–63.
[557] Lee JS, Pande GN. A new joint element for the analysis of media having discrete discontinuities. Mech Cohes-Frict Mater 1999;4:487–504.
[558] Pariseau WG. An equivalent plasticity theory for jointed rock masses. Int J Rock Mech Min Sci 1999;36:907–18.
[559] Kachanov LM. Time of the rupture process under creep conditions. Izv Akad Nauk SSR Otd Tekh Nauk 1958;8:26–31.
[560] Oliver J. On the discrete constitutive models induced by strong discontinuity kinematics and continuum constitutive equations. Int J Solids Struct 2000;37:7207–29.
[561] Oliver J, Huespe AE, Pulido MDG, Chaves E. From continuum mechanics to fracture mechanics: the strong discontinuity approach. Eng Fract Mech 2002;69:113–36.
[562] Krajcinovic D. Damage mechanics: accomplishments, trends and needs. Int J Solids Struct 2000;37(1–2):267–77.
[563] de Borst R. Fracture in quasi-brittle materials: a review of continuum damage-based approaches. Eng Fract Mech 2002;26:95–112.
[564] Ichikawa Y, Ito T, Mróz Z. A strain localization condition applying multi-response theory. Ing-Arch 1990;60:542–52.
[565] Kawamoto T, Ichikawa Y, Kyoya T. Deformation and fracturing behaviour of discontinuous rock mass and damage mechanics theory. Int J Numer Anal Methods Geomech 1988;12:1–30.
[566] Aubertin M, Simon R. A damage initiation criterion for low porosity rocks. Int J Rock Mech Min Sci 1997;34(3/4):554 (Paper No. 17).

[567] Grabinsky MW, Kamaleddine FF. Numerical analysis of an extended arrangement of periodic discrete fractures. Int J Rock Mech Min Sci 1997;34(3/4):533 (Paper No. 109).
[568] Shao JF. Poroelastic behaviour of brittle rock materials with anisotropic damage. Mech Mater 1998;30:41–53.
[569] Shao JF, Rudnicki JW. A microcrack-based continuous damage model for brittle geomechanics. Mech Mater 2000;32:607–19.
[570] Yang C, Daemen JJK. Temperature effects on creep of Tuff and its time-dependent damage analysis. Int J Rock Mech Min Sci 1997;34(3/4):383–4 (Paper No. 345).
[571] Chazallon A, Hicher PY. A constitutive model coupling elastoplasticity and damage for cohesive-frictional materials. Mech Cohes-Frict Mater 1998;3:41–63.
[572] Homand-Etienne F, Hoxha D, Shao JF. A continuum damage constitutive law for brittle rocks. Comput Geosci 1998;22(2): 135–51.
[573] Basista M, Gross D. The sliding crack model of brittle deformation: an internal variable approach. Int J Solids Struct 1998;35(5/6):487–509.
[574] Zhao Y. Crack pattern evolution and a fractal damage constitutive model for rock. Int J Rock Mech Min Sci 1998; 35(3):349–66.
[575] Carmelier J. Optimal estimation of gradient damage parameters from localization phenomena in quasi-brittle materials. Mech Cohes-Frict Mater 1999;4:1–16.
[576] Chen EP. Non-local effects on dynamic damage accumulation in brittle solids. Int J Numer Anal Methods Geomech 1999;23: 1–21.
[577] Dragon A, Halm D, Désoyer Th. Anisotropic damage in quasi-brittle solids: modelling, computational issues and applications. Comput Methods Appl Mech Eng 2000;183:331–52.
[578] Jessell M, Bons P, Evans L, Barr T, Stüwe K. Elle: the numerical simulation of metamorphic and deformation microstructures. Comput Geosci 2001;27:17–30.
[579] Li N, Chen W, Zhang P, Swoboda S. The mechanical properties and a fatigue-damage model for jointed rock masses subjected to dynamic cyclical loading. Int J Rock Mech Min Sci 2001; 38:1071–9.
[580] Brencich A, Gambarotta L. Isotropic damage model with different tensile-compressive response for brittle materials. Int J Solids Struct 2000;38:5865–92.
[581] Souley M, Homand F, Pepa S, Hoxha D. Damage-induced permeability changes in granite: a case example at the uRL in Canada. Int J Rock Mech Min Sci 2001;38:297–310.
[582] Comi C. Computational modelling of gradient-enhanced damage in quasi-brittle materials. Mech Cohes-Frict Mater 1999;4:17–36.
[583] Comi C. A non-local model with tension and compression damage mechanisms. Eur J Mech A/Solids 2001;20:1–22.
[584] Peerlings RH, de Borst R, Brekemans WA, Geers MGD. Localization issues in local and non-local continuum approaches to fracture. Eur J Mech A/Solids 2002;21:175–89.
[585] Pisarenko D, Gland N. Modeling of scale effects of damage in cemented granular rocks. Phys Chem Earth (A) 2001;26(1–2): 83–8.
[586] Yang R, Bawden WF, Katsabanis PD. A new constitutive model for blast damage. Int J Rock Mech Min Sci 1996;33(3):245–54.
[587] Liu L, Katsabanis PD. Development of a continuum damage model for blasting analysis. Int J Rock Mech Min Sci 1997;34(2):217–31.
[588] Rossmanith HP, Uenishi K. Post-blast bench block stability assessment. Int J Rock Mech Min Sci 1997;34(3/4):627 (Paper No. 264).
[589] Ma GW, Hao H, Zhou YX. Modeling of wave propagation induced by underground explosion. Comput Geotech 1998;22 (3/4):283–303.
[590] Wu C, Hao H, Zhou Y. Fuzzy-random probabilistic analysis of rock mass response to explosive loads. Comput Geotech 1999; 25:205–25.
[591] Desai CS, Zhang W. Computational aspects of disturbed state constitutive models. Comput Methods Appl Mech Eng 1998; 151:361–76.
[592] Sellers E, Scheele F. Prediction of anisotropic damage in experiments simulating mining in Witwatersrand quartzite blocks. Int J Rock Mech Min Sci Geomech Abstr 1996;33(7): 659–70.
[593] Cerrolaza M, Garcia R. Boundary elements and damage mechanics to analyze excavations in rock mass. Eng Anal Boundary Elements 1997;20:1–16.
[594] Elata D. Modeling wellbore breakouts. Int J Rock Mech Min Sci 1997;34(3/4):419 (Paper No. 72).
[595] Nazimko VV, Peng SS, Lapteev A, Alexandrov S, Sazhnev V. Damage mechanics around a tunnel due to incremental ground pressure. Int J Rock Mech Min Sci 1997;34(3/4):655 (Paper No. 222).
[596] Nazimko VV, Lapteev AA, Sazhnev VP. Rock mass self-supporting effect utilization for enhancement stability of a tunnel. Int J Rock Mech Min Sci 1997;34(3/4):657 (Paper No. 223).
[597] Swoboda G, Shen XP, Rosas L. Damage model for jointed rock mass and its application to tunnelling. Comput Geosci 1998; 22(3/4):183–203.
[598] Zhu W, Li L. Optimizing the construction sequence of a series of underground opening using dynamic construction mechanics and a rock mass fracture damage model. Int J Rock Mech Min Sci 2000;37:517–23.
[599] Stephansson O, editor. Fundamentals of rock joints. Proceedings of the International Symposium on Fundamentals of Rock Joints, Björkliden, Sweden, 15–20 September 1985. Luleä, Sweden: CENTIK publishers, 1985.
[600] Barton N, Stephansson O, editors. Rock joints. Proceedings of International Symposium On rock joints, Loen, Norway, 4–6 June 1990. Rotterdam: Balkema, 1990.
[601] Myer LR, Cook NGW, Goodman RE, Tsang CF, editors. Fractured and jointed rock masses. Proceedings of the conference on fractured and jointed rock masses, Lake Tahoe, USA, 3–5 June 1992. Rotterdam: Balkema, 1995.
[602] Rossmanith HP, editor. Mechanics of jointed and faulted rock. Proceedings of the First International Conference on the mechanics of jointed and faulted rock-MJFR-1 Vienna, 18–20 April 1990. Rotterdam: Balkema, 1990.
[603] Rossmanith HP, editor. Mechanics of jointed and faulted rock. Proceedings of the Second International Conference on the Mechanics of Jointed and Faulted Rock-MJFR-2, Vienna, 10–14 April 1995. Rotterdam: Balkema, 1995.
[604] Rossmanith HP, editor. Mechanics of jointed and faulted rock. Proceedings of the Third International Conference on the mechanics of jointed and faulted rock-MJFR-3, Vienna, 6–9 April 1998. Rotterdam: Balkema, 1998.
[605] Chernyshev SN, Dearman WR. Rock fractures. London: Butterworth-Heinemann Ltd, 1991.
[606] Lee CH, Farmer I. Fluid flow in discontinuous rocks. London: Chapman & Hall, 1993.
[607] Selvadurai APS, Boulon MJ, editors. Mechanics of Geomaterial interfaces. Amsterdam: Elsevier, 1995.
[608] Hudson JA, editor in-chief. Comprehensive rock engineering, vols. 1–5. Oxford: Pergamon Press, Elsevier, 1993. 4407p.
[609] Indrarantna B, Haque A. Shear behaviour of rock joints. Rotterdam: Balkema, 2000.
[610] Indrarantna B, Ranjith PG. Hydromechanical aspects and unsaturated flow in jointed rock. Rotterdam: Balkema, 2001.

[611] Jing L, Stephansson O. Mechanics of rock joints: experimental aspects. In: Selvadurai APS, Boulon M, editors. Mechanics of geomaterial interfaces. Amsterdam: Elsevier, 1995. p. 317–42.
[612] Ohnishi Y, Chan T, Jing L. Constitutive models of rock joints. In: Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media, Developments in Geotechnical Engineering, vol. 79. Amsterdam: Elsevier, 1996. p. 57–92.
[613] Maksimovic M. A family of non-linear failure envelopes for non-cemented soil and rock discontinuities. EJGE (Electron J Geotech Eng), 1996, Paper 1996-07.
[614] Kaliakin VN, Li J. Insight into deficiencies associated with commonly used zero-thickness interface elements. Comput Geotech 1995;17:225–52.
[615] Day RA, Potts DM. Zero thickness interface elements-numerical stability and applications. Int J Numer Anal Methods Geomech 1994;18(10):689–708.
[616] Bandis S, Lundsen AC, Barton NR. Fundamentals of rock joint deformation. Int J Rock Mech Min Sci Geomech Abstr 1983;20(6):249–68.
[617] Barton N, Bandis S, Bakhtar K. Strength, deformation and conductivity coupling of rock joints. Int J Rock Mech Min Sci Geomech Abstr 1985;22(3):121–40.
[618] Dong JJ, Pan YW. A hierarchical model of rough rock joints based on micromechanics. Int J Rock Mech Min Sci 1996; 33(2):111–23.
[619] Desai CS, Ma Y. Modeling of joints and interfaces using the disturbed state concept. Int J Numer Anal Methods Geomech 1992;16:623–53.
[620] Desai CS. Hierarchical single surface and the disturbed state constitutive models with emphasis on geotechnical applications. In: Saxena KR, editor. Geotechnical engineering: emerging trends in design and practice. Rotterdam: Balkema, 1994. p. 115–54.
[621] Amadei B, Saeb S. Constitutive models of rock joints. In: Barton N, Stephansson O, editors. Rock joints. Proceedings of the International Symposium On rock joints, Loen, Norway, 4–6 June 1990. Rotterdam: Balkema, 1990. p. 581–94.
[622] Souley M, Homand F, Amadei B. An extension of the Saeb and Amadie constitutive model for rock joints to include cyclic loading paths. Int J Rock Mech Min Sci Geomech Abstr 1985;32(2):101–9.
[623] Fishman KL, Desai CS. A constitutive model for hardening behaviour of rock joints. In: Desai CS, editor. Constitutive laws for engineering materials: theory and applications, 1987. p. 1043–50.
[624] Plesha ME. Constitutive models for rock discontinuities with dilatancy and surface degradation. Int J Numer Anal Methods Geomech 1987;11:345–62.
[625] Nguyen TS, Selvadurai APS. A model for coupled mechanical and hydraulic behaviour of a rock joint. Int J Numer Anal Methods Geomech 1998;22:29–48.
[626] Lee HS, Park YJ, Cho TF, You KH. Influence of asperity degradation on the mechanical behavior of rough rock joint under cyclic shear loading. Int J Rock Mech Min Sci 2001;38: 967–80.
[627] Plesha ME, Ni D. Scaling of geological discontinuity normal load-deformation response using fractal geometry. Int J Numer Anal Methods Geomech 2001;25:741–56.
[628] Lee HS, Cho TF. Hydraulic characteristics of rough fractures in linear flow under normal and shear load. Rock Mech Rock Eng 2002;35(4):299–318.
[629] Desai CS, Fishman KL. Plasticity-based constitutive model with associated testing for joints. Int J Rock Mech Min Sci Geomech Abstr 1991;28(1):15–26.
[630] Desai CS, Samtani NC, Vulliet L. Constitutive modelling and analysis of creeping slopes. J Geotech Eng ASCE 1995;12(1): 43–56.
[631] Mróz Z, Giambanco G. In interface model for analysis of deformation behaviour of discontinuities. Int J Numer Anal Methods Geomech 1996;20(1):1–33.
[632] Lespinasse M, Sausse J. Quantification of fluid flow: hydro-mechanical behaviour of different natural rough surfaces. J Geotech Explor 2000;69–70:483–6.
[633] Swan G. Determination of stiffness and other joint properties from roughness measurements. Rock Mech Rock Eng 1983; 16:16–38.
[634] Sun Z, Gerrard C, Stephansson O. Rock joint compliance tests for compression and shear loads. Int J Rock Mech Min Sci Geomech Abstr 1985;22(4):197–213.
[635] Swan G, Sun Z. Prediction of shear behaviour of joints using profiles. Rock Mech Rock Eng 1985;18:183–212.
[636] Yoshioka N, Scholz CH. Elastic properties of contacting surfaces under normal and shear loads. 1. Theory. J Geophys Res 1989;94:17681–90.
[637] Yoshioka N, Scholz CH. Elastic properties of contacting surfaces under normal and shear loads. 2. Comparison of theory with experiment. J Geophys Res 1989;94:17691–700.
[638] Lei XY, Swoboda G, Zenz G. Application of contact–friction interface element to tunnel excavation in faulted rock. Comput Geotech 1995;17:349–70.
[639] Greenwood JA, Williamson JBP. Contact of nominally flat rough surfaces. Proc R Soc London A 1966;295:300–19.
[640] Greenwood JA, Tipp JH. The contact of two nominally flat rough surfaces. Proc Inst Mech Eng Part C 1971;185:625–33.
[641] Barton N. Review of a new shear strength criterion for rock joints. Eng Geol 1973;7:287–332.
[642] Barton N, Choubey V. The shear strength of rock joints in theory and practice. Rock Mech 1976;10:1–54.
[643] Kwaśniewski MA, Wang JA. Surface roughness evolution and mechanical behaviour of rock joints under shear. Int J Rock Mech Min Sci 1997;34(3/4):709 (Paper No. 157).
[644] Panagouli OK, Mistakidis ES, Panagiotopoulos PD. On the fractal fracture in brittle structures-numerical approach. Comput Methods Appl Mech Eng 1997;147:1–15.
[645] Homand F, Belem T, Souley M. Friction and degradation of rock joint surfaces under shear loads. Int J Numer Anal Methods Geomech 2001;25:973–99.
[646] Roko RO, Daemen JJK, Myers DE. Variogram characterization of joint surface morphology and asperity deformation during shearing. Int J Rock Mech Min Sci 1997;34(1):71–84.
[647] Yang ZY, Chen GL. Application of the self-affinity concept to the scale effect of joint roughness. Rock Mech Rock Eng 1999;32(3):221–9.
[648] Belem T, Homand-Etienne F, Souley M. Quantitative parameters for rock joint surface roughness. Rock Mech Rock Eng 2000;33(4):217–42.
[649] Yang ZY, Lo SC, Di CC. Reassessing the joint roughness coefficient (JRC) estimation using Z2. Rock Mech Rock Eng 2001;34(3):243–51.
[650] Yang ZY, Di CC, Lo SC. Two-dimensional Hurst index of joint surfaces. Rock Mech Rock Eng 2001;34(4):323–45.
[651] Yang ZY, Di CC, Yen KC. The effect of asperity order on the roughness of rock joints. Int J Rock Mech Min Sci 2001;38: 745–52.
[652] Lanaro F. A random field model for surface roughness and aperture of rock fractures. Int J Rock Mech Min Sci 2000;37:1195–210.
[653] Fu H, Li L, Liu B, Hou Z, Lux KH. Application of fractal theory in analyzing character of joints and cracks inside a rockmass. EJGE (Electron J Geotech Eng), 2001, Paper No. 2001-6.

[654] Whitehouse DJ. Fractal or fiction. Wear 2001;249:345–53.
[655] Gangi AF. Variation of whole and fractured porous rock permeability with confining pressure. Int J Rock Mech Min Sci Geomech Abstr 1978;15:249–57.
[656] Walsh JB, Grosenbaugh MA. A new model for analysing the effect of fractures on compressibility. J Geophys Res 1979;84:3532–6.
[657] Witherspoon PA, Wang JSY, Iwai K, Gale JE. Validity of cubic law for fluid flow in a deformable rock fracture. Water Resour Res 1980;16(6):1016–24.
[658] Walsh JB. Effect of pore pressure and confining pressure on fracture permeability. Int J Rock Mech Min Sci Geomech Abstr 1981;18:429–35.
[659] Tsang YW, Witherspoon PA. Hydromechanical behaviour of a deformable rock fracture subject to normal stress. J Geophys Res 1986;86(B10):9287–98.
[660] Raven KG, Gale JE. Water flow in a natural rock fracture as a function of stress and sample size. Int J Rock Mech Min Sci Geomech Abstr 1985;22(4):251–61.
[661] Brown SR, Scholz CH. Closure of random elastic surfaces in contact. J Geophys Res 1985;90:5531–45.
[662] Brown SR. Fluid flow through rock joints: the effect of surface roughness. J Geophys Res 1987;92:1337–47.
[663] Pyrak-Nolte LJ, Cook NGW. Fluid percolation through single fractures. Geophys Res Lett 1988;15(11):1247–50.
[664] Cook NGW. Natural joints in rock: mechanical, hydraulic and seismic behaviour and properties under normal stress. The First Jaeger Memorial Lecture, the 29th US Rock Mechanics Symposium, Minneapolis, 1988.
[665] Olsson WA. The effects of slip on the flow of fluid through a fracture. Geophys Res Lett 1992;19:541–3.
[666] Olsson WA, Brown SR. Hydromechanical response of a fracture undergoing compression and shear. Int J Rock Mech Min Sci Geomech Abstr 1993;30(7):845–51.
[667] Ng KLA, Small JC. Behaviour of joints and interfaces subjected to water pressure. Comput Geotech 1997;20(1):71–93.
[668] Oron A, Berkowitz B. Flow in rock fractures: the local cubic law assumption re-examined. Water Resour Res 1998;34(11):2811–25.
[669] Power WL, Durham WB. Topography of natural and artificial fractures in granitic rocks: implications for studies of rock friction and fluid migration. Int J Rock Mech Min Sci 1997;34(6):979–89.
[670] Nicholl MJ, Rajaram H, Glass RJ, Detwiller R. Saturated flow in a single fracture: evaluation of the Reynolds equation in measured aperture fields. Water Resour Res 1999;35(11):3361–73.
[671] Yeo IW, de Freitas MH, Zimmerman RW. Effect of shear displacement on the aperture and permeability of a rock fracture. Int J Rock Mech Min Sci 1998;35(8):1051–70.
[672] Indraratna B, Ranjith PG, Gale W. Single phase water flow through rock fractures. Geotech Geol Eng 1999;17:211–40.
[673] Pyrak-Nolte LJ, Morris JP. Single fracture under normal stress: the relation between fracture specific stiffness and fluid flow. Int J Rock Mech Min Sci 2000;37:245–62.
[674] Olsson R, Barton N. An improved model for hydromechanical coupling during shearing of rock joints. Int J Rock Mech Min Sci 2001;38:317–29.
[675] Fakharian K, Evgin E. Elasto-plastic modelling of stress-path-dependent behaviour of interfaces. Int J Numer Anal Methods Geomech 2000;24:183–99.
[676] Cox JV, Herrmann L. Development of a plasticity bond model for steel reinforcement. Mech Cohes-Frict Mater 1998;3:155–80.
[677] Cox JV, Herrmann L. Validation of a plasticity bond model for steel reinforcement. Mech Cohes-Frict Mater 1999;4:361–89.
[678] Tsang CF, editor. Coupled processes associated with nuclear waste repositories. New York: Academic Press, 1987.
[679] Tsang CF. Coupled thermomechanical and hydrochemical processes in tock fractures. Rev Geophys 1991;29:537–48.
[680] Hudson JA. Rock engineering systems: theory and practice. Chichester: Ellis Horwood.
[681] Jiao Y, Hudson JA. Fully coupled model for rock engineering system. Int J Rock Mech Min Sci Geomech Abstr 1995;32(5):491–512.
[682] von Terzaghi K. Die berechnug der Durchlässigkeitsziffer des Tones aus dem Verlauf der hydrodynamischen Spannungserscheinungen. Sitzungsber Akad Wiss Math-Naturwiss Section IIa 1923;132(3/4):125–38.
[683] Biot MA. General theory of three-dimensional consolidation. J Appl Phys 1941;12:155–64.
[684] Biot MA. General solution of the equation of elasticity and consolidation for a porous material. J Appl Mech 1956;23:91–6.
[685] Morland LW. A simple constitutive theory for fluid saturated porous solids. J Geophys Res 1972;77:890–900.
[686] Bowen RM. Compressible porous media models by use of theories of mixtures. Int J Eng Sci 1982;20:697–735.
[687] Hassanizadeh M, Gray WG. General conservation equations for multiphase systems: 1. Averaging procedures. Adv Water Res 1979;2:131–44.
[688] Hassanizadeh M, Gray WG. General conservation equations for multiphase systems: 2. Mass momenta, energy and entropy equations. Adv Water Res 1979;2:191–203.
[689] Hassanizadeh M, Gray WG. General conservation equations for multiphase systems: 3. Constitutive theory for porous media flow. Adv Water Res 1980;3:25–40.
[690] Hassanizadeh M, Gray WG. Mechanics and theormodynamics of multiphase flow in porous media including interphase transport. Adv Water Res 1990;13:169–86.
[691] Achanta S, Cushman JH, Okos MR. On multicomponent, multiphase thermomechanics with interfaces. Int J Eng Sci 1994;32:1717–38.
[692] de Boer R. The thermodynamic structure and constitutive equations for fluid-saturated compressible and incompressible elastic porous solids. Int J Solids Struct 1998;35(34–35):4557–73.
[693] Whitaker S. Simultaneous heat, mass and momentum transfer in porous media: a theory of drying. New York: Academic Press, 1977.
[694] Domenico PA, Schwartz FW. Physical and chemical hydrogeology. New York: Wiley, 1990.
[695] Charlez PA. Rock mechanics. Vol. 1—theoretical fundamentals. Paris: Editions Technip., 1991.
[696] Charlez P, Keramsi D, editors. Mechanics of porous media. Lecture notes of the Mechanics of Porous Media summer school, June 1994. Rotterdam: Balkema, 1995.
[697] Coussy O. Mechanics of porous media. Chichester: Wiley, 1995.
[698] Selvadurai APS, editor. Mechanics of poroelastic media. Dordrecht: Kluwer Academic Publishers, 1996.
[699] Lewis RW, Schrefler BA. The finite element method in the static and dynamic deformation and consolidation of porous media, 2nd ed. Chichester: Wiley, 1998.
[700] Lewis RW, Schrefler BA. The finite element method in the deformation and consolidation of porous media. Chichester: Wiley, 1987.
[701] Bai M, Elsworth D. Coupled processes in subsurface deformation, flow and transport. Reston, VA: ASCE Press, 2000.
[702] Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media. Rotterdam: Elsevier, 1996.

[703] Noorishad J, Tsang CF, Witherspoon PA. Theoretical and field studies of coupled hydromechanical behaviour of fractured rocks—1. Development and verification of a numerical simulator. Int J Rock Mech Min Sci Geomech Abstr 1992;29(4):401–9.

[704] Noorishad J, Tsang CF. Coupled thermohydroelasticity phenomena in variably saturated fractured porous rocks—formulation and numerical solution. In: Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media. Rotterdam: Elsevier, 1996. p. 93–134.

[705] Rutqvist J, Börgesson L, Chijimatsu M, Kobayashi A, Jing L, Nguyen TS, Noorishad J, Tsang CF. Thermohydro-mechanics of partially saturated geological media: governing equations and formulation of four finite element models. Int J Rock Mech Min Sci 2001;38(1):105–27.

[706] Rutqvist J, Börgsson L, Chijimatsu M, Nguyen TS, Jing L, Noorishad J, Tsang CF. Coupled thermo-hydro-mechanical analysis of a heater test in fractured rock and bentonite at Kamaishi Mine—comparison of field results to predictions of four finite element codes. Int J Rock Mech Min Sci 2001;38(1): 129–42.

[707] Börgesson L, Chijimatsu M, Fujita T, Nguyen TS, Rutqvist J, Jing L. Thermo-hydro-mechanical characterization of a bentonite-based buffer material by laboratory tests and numerical back analysis. Int J Rock Mech Min Sci 2001;38(1):95–104.

[708] Nguyen TS, Börgesson L, Chijimatsu M, Rutqvist J, Fujita T, Hernelind J, Kobayashi A, Ohnishi Y, Tanaka M, Jing L. Hydro-mechanical response of a fractured granite rock mass to excavation of a test pit—the Kamaishi Mine experiments in Japan. Int J Rock Mech Min Sci 2001;38(1):79–94.

[709] Preuss K. TOUGH2—a general purpose numerical simulator for multiphase fluid and heat flow. Lawrence Berkeley Laboratory Report LBL-29400, Berkeley, CA, 1991.

[710] Millard A. Short description of CASTEM 2000 and TRIO-EF. In: Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media. Rotterdam: Elsevier, 1996. p. 559–64.

[711] Ohnishi Y, Kobayashi A. THAMES. In: Stephansson O, Jing L, Tsang CF, editors. Coupled thermo-hydro-mechanical processes of fractured media. Rotterdam: Elsevier, 1996. p. 545–9.

[712] Schrefler BA. Computer modelling in environmental geomechanics. Comput Struct 2001;79:2209–23.

[713] Abdaliah G, Thoraval A, Sfeir A, Piguet JP. Thermal convection of fluid in fractured media. Int J Rock Mech Min Sci Geomech Abstr 1995;32(5):481–90.

[714] Jing L, Stephansson O, Tsang CF, Kautsky F. DECOVALEX—mathematical models of coupled T–H–M processes for nuclear waste repositories. Executive summary for Phases I, II and III. SKI Report 96:58. Swedish Nuclear Power Inspectorate, Stockholm, Sweden, 1996.

[715] Jing L, Stephansson O, Tsang CF, Knight LJ, Kautsky F. DECOVALEX II project, executive summary. SKI Report 99:24. Swedish Nuclear Power Inspectorate, Stockholm, Sweden, 1999.

[716] Stephansson O. Introduction. Special issue for thermo-hydro-mechanical coupling in rock mechanics. Int J Rock Mech Min Sci 1995;32(5); p. 387, Pergamon.

[717] Stephansson O, Tsang CF, Kautsky F. Foreword. Special issue for thermo-hydro-mechanical coupling in rock mechanics. Int J Rock Mech Min Sci 2001;38(1); p. 1–4, Pergamon.

[718] Zhao C, Hobbs BE, Baxter K, Mühlhaus HB, Ord A. A numerical study of pore-fluid, thermal and mass flow in fluid-saturated porous rock basins. Eng Comput 1999;16(2):202–14.

[719] Yang XS. A unified approach to mechanical compaction, pressure solution, mineral reactions and the temperature distribution in hydrocarbon basins. Tectonophysics 2001;330: 141–51.

[720] Sasaki T, Morikawa S. Thermo-mechanical consolidation coupling analysis on jointed rock mass by the finite element method. Eng Comput 1996;13(7):70–86.

[721] Gawin D, Schrefler BA. Thermo-hydro-mechanical analysis of partially saturated materials. Eng Comput 1996;13(7): 113–43.

[722] Wang X, Schrefler BA. A multi-frontal parallel algorithm for coupled thermo-hydro-mechanical analysis of deformable porous media. Int J Numer Methods Eng 1998;43:1069–83.

[723] Cervera M, Codina R, Galindo M. On the computational efficiency and implementation of block-iterative algorithms for non-linear coupled problems. Eng Comput 1996;13(6):4–30.

[724] Thomas HR, Yang HT, He Y. A sub-structure based parallel solution of coupled thermo-hydro-mechanical modeling of unsaturated soil. Eng Comput 1999;16(4):428–42.

[725] Thomas HR, Missoum H. Three-dimensional coupled heat, moisture and air transfer in a deformable unsaturated soil. Int J Numer Methods Eng 1999;44:919–43.

[726] Thomas HR, Cleall PJ. Inclusion of expansive clay behaviour in coupled thermo hydraulic mechanical models. Eng Geol 1999;54:93–108.

[727] Selvadurai APS, Nguyen TS. Mechanics and fluid transport in a degradable discontinuity. Eng Geol 1999;53:243–9.

[728] Selvadurai APS, Nguyen TS. Scoping analyses of the coupled thermal-hydrological-mechanical behaviour of the rock mass around a nuclear fuel waste repository. Eng Geol 1996;47: 379–400.

[729] Hudson JA, Stephansson O, Andersson J, Jing L. Coupled T-H-M issues relating to radioactive waste repository design and performance. Int J Rock Mech Min Sci 2001;38(1):143–61.

[730] Nithiarasu P, Sujatha KS, Ravindran K, Sundararajan T, Seetharamu KN. Non-Darcy natural convection in a hydrodynamically and thermally anisotropic porous medium. Comput Methods Appl Mech Eng 2000;188:413–30.

[731] Zimmerman RW. Coupling in poroelasticity and thermoelasticity. Int J Rock Mech Min Sci 2000;37(1):79–87.

[732] Lai YM, Wu ZW, Zhu YL, Zhu LN. Nonlinear analysis for the coupled problem of temperature, seepage and stress fields in cold-region tunnels. Tunnelling Underground Space Technol 1998;13(4):435–40;
Sakurai S. Direct strain evaluation technique in construction of underground openings. Proceedings of the 22nd US Symposium Rock Mechanics. Cambridge, MA: MIT, 1981. p. 278–82.

[733] Sakurai S, Akutagawa S. Some aspects of back analysis in geotechnical engineering. EUROC '93, Lisbon, Portugal. Rotterdam: Balkema, 1995. p. 1130–40.

[734] Gens A, Ledesma A, Alonso EE. Estimation of parameters in geotechnical backanalysis. II—application to a tunnel problem. Comput Geotech 1996;18(1):29–46.

[735] Ledesma A, Gens A, Alonso EE. Estimation of parameters in geotechnical backanalysis—I. Maximum likelihood approach. Comput Geotech 1996;18(1):1–27.

[736] Ledesma A, Gens A, Alonso EE. Parameter and variance estimation in geotechnical back-analysis using prior information. Int J Numer Anal Methods Geomech 1996;20(2):119–41.

[737] Mello Franco JA, Assis AP, Mansur WJ, Telles JCF, Santiago AF. Design aspects of the underground structures of the Serra da Mesa hydroelectric power plant. Int J Rock Mech Min Sci 1997;34(3/4):580, Paper No. 16.

[738] Hojo A, Nakamura M, Sakurai S, Akutagawa S. Back analysis of non-elastic strains within a discontinuous rock mass around a large underground power house cavern. Int J Rock Mech Min Sci 1997;34(3/4):570, Paper No. 8.

[739] Sakurai S. Lessons learned from field measurements in tunnelling. Tunnelling Underground Space Technol 1997;12(4): 453–60.

[740] Singh B, Viladkar MN, Samadhiya NK, Mehrotra VK. Rock mass strength parameters mobilised in tunnels. Tunnelling Underground Space Technol 1997;12(1):47–54.

[741] Pelizza S, Oreste PP, Pella D, Oggeri C. Stability analysis of a large cavern in Italy for quarrying exploitation of a pink marble. Tunnelling Underground Space Technol 2000;15(4):421–35.

[742] Yang Z, Lee CF, Wang S. Three-dimensional back-analysis of displacements in exploration adits-principles and application. Int J Rock Mech Min Sci 2000;37:525–33.

[743] Krajewski W, Edelmann L, Plamitzer R. Ability and limits of numerical methods for the design of deep construction pits. Comput Geotech 2001;28:425–44.

[744] Chi SY, Chern JC, Lin CC. Optimized back-analysis for tunnelling-induced ground movement using equivalent ground loss model. Tunnelling Underground Space Technol 2001;16: 159–65.

[745] Ai-Homoud AS, Tai AB, Taqieddin SA. A comparative study of slope stability methods and mitigative design of a highway embankment landslide with a potential for deep seated sliding. Eng Geol 1997;47(1–2):157–73.

[746] Okui Y, Tokunaga A, Shinji M, Mori M. New back analysis method of slope stability by using field measurements. Int J Rock Mech Min Sci 1997;34(3/4):515, Paper No. 234.

[747] Sonmez H, Ulusay R, Gokceoglu C. A practical procedure for the back analysis of slope failure in closely jointed rock. Int J Rock Mech Min Sci 1998;35(2):219–33.

[748] Yang L, Zhang K, Wang Y. Back analysis of initial rock stress and time-dependent parameters. Int J Rock Mech Min Sci Geomech Abstr 1996;33(6):641–5.

[749] Obara Y, Nakamura N, Kang SS, Kaneko K. Measurement of local stress and estimation of regional stress associated with stability assessment of an open-pit rock slope. Int J Rock Mech Min Sci 2000;37(8):1211–21.

[750] Guo WD. Visco-elastic consolidation subsequent to pile installation. Comput Geotech 2000;26(2):113–44.

[751] Kim YT, Lee SR. An equivalent model and back-analysis technique for modelling in situ consolidation behaviour of drainage-installed soft deposits. Comput Geotech 1997;20(2):125–42.

[752] de Marsily G, Delhomme JP, Delay F, Buoro A. 40 years of inverse solution in hydrogeology. Earth Planet Sci 1999;329: 73–87 (in French, with an English summary).

[753] Chen J, Hopmans JW, Grismer ME. Parameter estimation of two-fluid capillary pressure-saturation and permeability function. Adv Water Res 1999;22(5):479–93.

[754] Fatullayev A, Can E. Numerical procedure for identification of water capacity of porous media. Math Comput Simulations 2000;52:113–20.

[755] Finsterle S, Faybishenko B. Inverse modelling of a radial multistep outflow experiment for determining unsaturated hydraulic properties. Adv Water Res 1999;22(5):431–44.

[756] Hanna S, Jim Yeh TC. Estimation of co-conditional moments of transmissivity, hydraulic head and velocity fields. Adv Water Res 1998;22(1):87–95.

[757] Jhorar RK, Bastiaanssen WGM, Fesses RA, van Dam JC. Inversely estimating soil hydraulic functions using evapotranspiration fluxes. J Hydrol 2002;258:198–213.

[758] Katsifarakis KL, Karpouzos DK, Theodossiou N. Combined use of BEM and genetic algorithms in groundwater flow and mass transport problems. Eng Anal Boundary Elements 1999;23: 555–65.

[759] Vasco DW, Karasaki K. Inversion of pressure observations: an integral formulation. J Hydrol 2001;253:27–40.

[760] Lesnic D, Elliott L, Ingham DB, Clennell B, Knipe RJ. A mathematical model and numerical investigation for determining the hydraulic conductivity of rocks. Int J Rock Mech Min Sci 1997;34(5):741–59.

[761] Li H, Yang Q. A least-square penalty method algorithm for inverse problems of steady-state aquifer models. Adv Water Res 2000;23:867–80.

[762] Mayer AS, Huang C. Development and application of a coupled-process parameter inversion model based on the maximum likelihood estimation method. Adv Water Res 1999; 22(8):841–53.

[763] Nützmann G, Thiele M, Maciejewski S, Joswig K. Inverse modelling techniques for determining hydraulic properties of coarse-textured porous media by transient outflow methods. Adv Water Res 1998;22(3):273–84.

[764] Russo D. On the estimation of parameters of log-unsaturated conductivity covariance from solute transport data. Adv Water Res 1997;20(4):191–205.

[765] Roth C, Chilès JP, de Fouquet C. Combining geostatistics and flow simulators to identify transmissivity. Adv Water Res 1998; 21:555–65.

[766] Wen XH, Capilla JE, Deutsch CV, Gómez-Hernandez JJ, Cullick AS. A program to create permeability fields that honour single-phase flow rate and pressure data. Comput Geosci 1999; 25:217–30.

[767] Wen XH, Deutsch CV, Cullick AS. Construction of geostatistical aquifer models integrating dynamic flow and tracer data using inverse technique. J Hydrol 2002;255:151–68.

[768] Wang PP, Zheng C. An efficient approach for successively perturbed groundwater models. Adv Water Res 1998;21: 499–508.

[769] Chapko R. On the numerical solution of direct and inverse problems for the heat equation in a semi-infinite region. J Comput Appl Math 1999;108:41–55.

[770] Hori M, Kameda T. Inversion of stress from strain without full knowledge of constitutive relations. J Mech Phys Solids 2001; 49:1621–38.

[771] Shaw J. Noniterative solution of inverse problems by the linear least square method. Appl Math Modelling 2001;25: 683–96.

[772] Barton N. Personal communications, 2001.

[773] Banks HT. Remarks on uncertainty assessment and management in modelling and computing. Math Comput Modelling 2001;33:39–47.