Numerical analysis of heat transfer in cooling of fish packages

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Abstract

The present work aims at finding the optimal finite difference scheme for the solution of problems involving pure heat transfer from the surface of solids suddenly exposed to a cooling environment. Fish samples in the form of infinite slab were considered, and a generalized mathematical model was constructed in dimensionless form. A more representative and accurate set of experimental data is chosen from the experimental work for comparison with the numerical results and evaluation of numerical schemes. In the analysis, a fully explicit finite difference scheme, an implicit finite difference scheme and different combination schemes with varying values of weighting factor are thoroughly studied. The characteristic dimension (half thickness of the slab) is divided into a number of divisions; \( n = 5, 10, 20, 30, 40, 50 \) and 100, respectively. All the possible options of the Fourier number increments are taken one by one to give the best convergence and minimal truncation error. The simplest explicit finite difference scheme with \( n = 10 \) and Fourier number increments one sixth of the square of the space division size appears to be highly reliable and accurate for such applications.

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1. Introduction

Cooling is widely used in a large variety of thermal applications ranging from food cooling to electronics cooling. Generally speaking, the refrigeration industry is extremely large, and plays one of the major roles in countries’ economies throughout the world [1].

Regardless of the type of cooling technique, knowledge and determination of heat transfer parameters especially in terms of the thermal diffusivity and heat transfer coefficient as well as estimation of temperature profiles are essential to attain an optimum cooling operation at the micro- and macro-scale application. Even small improvements in such heat
transfer applications during a cooling process may lead to significant amount of energy and financial savings. In practice, simple but accurate models for determining the heat transfer parameters are highly preferred by the cooling industry to complex methods. Accurate determination of such parameters for the objects subjected to cooling results in efficient energy use and optimum processing conditions [2].

In the thermal processes of perishable products, cooling is employed as one of the preservation techniques to prevent their spoilage and maintain their quality (e.g., self-life). The heat transfer taking place during such processes is essentially transient heat transfer, and analytical methods may not be sufficient for accurate solutions. In this regard, numerical techniques, e.g., either finite difference or finite element appear to be very powerful tools and are capable of handling any type of boundary conditions and product geometries. Any non-linearity or singularity can also be handled, and changes of thermophysical properties, if any, can easily be incorporated. Recently, many studies on the numerical analysis of heat transfer taking place during cooling and freezing processes of various food products and substances have been undertaken by various investigators [3–10]. In the present work the particular application of interest is the precooling of Malaysian Pangasius Sutchi fish packages as in cold storage and precooling. In fact, precooling is the first cooling operation of the perishable products in which their temperature is reduced to their storage temperature as soon as they are harvested or collected. The next step after the precooling is the cold storage [11,12]. Although this specie of fresh water fish is broadly used as a main food in Malaysia [13], no publications have been done regarding the thermal properties and heat transfer analysis during heating and cooling processes of such fish. In air blast cooling of canned food commodities, cooling occurs due to only convective heat transfer. Because of its relative simplicity, the finite difference method is more popularly used to solve the transient heat transfer problems related to food processes. By applying the numerical grid generation approach, it can be used for irregular geometry as effectively as the more complicated finite element method without sacrificing its simplicity.

A number of investigators have used finite difference methods for solving problems with pure convective heat transfer from the surface of food products [10,14–16]. Their models have given satisfactory results during air blast cooling of wrapped, packaged, tinned foods or during hydrocooling. Others researchers [17–19] have described several finite difference schemes for the solution of transient heat conduction equations with different initial and boundary conditions. Thibault [20] also reported a comprehensive research work in this regard.

The main objective of the present work is to conduct thorough comparative investigation of the fully explicit scheme, fully implicit scheme and different weighted averages of the two schemes so as to establish the scheme which is best suited for computing temperature–time variations during precooling of Pangasius Sutchi freshwater fish packages with pure convection heat transfer.

2. Mathematical modeling

The governing transient heat conduction equation in dimensionless form for isotropic solids in which heat transfer may be approximated to be unidirectional, with no internal heat generation, is written as

\[
\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau} \quad \text{for} \quad \tau \geq \tau_0 \quad 0 \leq X \leq 1
\]  

If the produce is initially at a uniform temperature and symmetrical cooling occurs, the initial and boundary conditions are defined, respectively, by the following equations:

\[
\theta = \theta(X) \quad \text{for} \quad \tau = \tau_0 \quad 0 \leq X \leq 1
\]  

\[
\frac{\partial \theta}{\partial X} = 0 \quad \text{for} \quad \tau > \tau_0 \quad X = 0
\]  

At the surface, the convection boundary condition is defined as:

\[
\frac{\partial \theta}{\partial X} = -Bi\theta \quad \text{for} \quad \tau > \tau_0 \quad X = 1
\]
The general finite difference representation of the governing heat conduction Eq. (1) is given [17] for infinite slab as follows:

$$\frac{\theta_{i}^{j+1} - \theta_{i}^{j}}{\Delta \tau} = \frac{\beta}{(\Delta X)^2} [\theta_{i-1}^{j+1} - 2\theta_{i}^{j+1} + \theta_{i+1}^{j+1}] + \frac{(1-\beta)}{(\Delta X)^2} [\theta_{i-1}^{j-1} - 2\theta_{i}^{j} + \theta_{i+1}^{j}]$$

(5)

Here,

$$\Delta X = \frac{1}{h}$$

(6)

where $\Delta \tau$= size of time step.

In implicit scheme the weighing factor $\beta$ is a real constant such that

$$0 \leq \beta \leq 1$$

(7)

For higher computational accuracy, the first derivatives in the center and surface boundary condition equations are written in the form of the four-point formulae [18], given, respectively, as follows:

$$\frac{\partial \theta}{\partial X} = \frac{1}{6\Delta X} \left( -11\theta_{0}^{j+1} + 18\theta_{1}^{j+1} - 9\theta_{2}^{j+1} + 2\theta_{3}^{j+1} \right)$$

for $X = 0$

(8)

$$\frac{\partial \theta}{\partial X} = \frac{1}{6\Delta X} \left( -2\theta_{n-1}^{j+1} + 9\theta_{n-2}^{j+1} - 18\theta_{n-3}^{j+1} + 11\theta_{n}^{j+1} \right)$$

for $X = 1$

(9)

These equations are based on Lagrangian interpolation and have a truncation error $O(\Delta X)^3$.

3. Experimental procedure

The experimental investigation was carried out on a slab-shaped sample of freshwater Pangasius Sutchi Malaysian fish. The work was started firstly with mass density measurement by means of electronic balance with resolution of 0.001g. The volume was measured by dipping the sample in a calibrated jar filled with water. The measurement of water content of the fish sample was made by a sensitive electronic balance fitted with infrared dryer set at 105°C for 12h. Mass of thinly cut fish pieces was determined before and after thorough drying until no further moisture loss was obtained. With the measured value of water mass fraction ($W$), its thermal conductivity was determined by Sweat’s correlation [21] as given below:

$$k = 0.08 + 0.52W$$

(10)

The specific heat was determined by Reidel’s model for fish meat above freezing point [22] and is given below:

$$C_p = 1.672 + 2.508W$$

(11)

The coefficient of heat transfer was calculated by $Nu$–$Re$ relationship [9]:

$$Nu = 0.668 Re^{0.5} Pr^{0.333}$$

(12)

An air-blast cooling duct, shown in Fig. 1, was designed and fabricated for the measurement of temperature–time records inside the fish flesh during its transient cooling. The test-rig consisted of a 4-m-long galvanized iron sheet air-duct of 0.33m×0.31m section, which was insulated with 15-mm-thick glass wool. The air was cooled by passing it over the cooling coils (precooler and recooler) of a R-22 refrigeration system. The temperature of the circulating air inside the test duct was maintained constant at 1°C. It was controlled through the adjustable pre-heater, heater, defrost heaters as well as by adjusting the evaporator pressure of the refrigeration system. The dampers $A$, $B$ and $C$ were used to control the velocity of air passing over the test container which was kept constant throughout the experiments at 5ms$^{-1}$.

The test container was designed in a form of rectangular (made from polystyrene) shape of dimensions of $2.54 \times 22.5 \times 22.5$ cm$^3$. The two major surfaces ($22.5 \times 22.5$ cm$^2$) were covered by copper sheets of 0.1 mm thickness. The four remainder minor surfaces ($2.54 \times 22.5$ cm$^2$) were surrounded by wooden sash to support the combination. A central cavity of dimensions $2.54 \times 2.54 \times 5.08$ cm$^3$ was implemented in the middle of the rectangular test container to
accommodate the fish sample. Fig. 2 reveals that the fish sample within the test container is surrounded by four insulated peripheral walls whereas the remaining opposite surfaces were in touch with the two copper covers to allow symmetrical one-dimensional heat transfer to take place. In order to fix the container inside the test duct, a pair of insulated hooks was

![Diagram of air blast cooling duct](image)

Fig. 1. Schematic diagram of air blast cooling duct.

![Cross-sectional view of the test container](image)

Fig. 2. Test container details.
attached to the inside of the upper surface of the test section. The test container was fastened to the upper hooks with the help of thin cotton threads to avoid heat conduction. The characteristic length, $X_0$, of the fish sample was half the thickness of test container (1.27 cm). Five copper–constantan thermocouple beads were installed inside the fish flesh, at the depths $X_0/5$, $2X_0/5$, $3X_0/5$, $4X_0/5$ and $X_0$ from the sample surface. In order to insert the temperature sensors at the desired depths, five fine holes were drilled at equal distances of 5 mm from each other at the middle of one copper sheet cover of the test container. The temperatures inside the fish flesh, and the dry bulb and wet bulb temperatures of the circulating air were measured with an accuracy of 0.1 °C with the type T copper–constantan thermocouples (T-TE/TE-24) with a wire diameter of 0.5 mm, and an operating temperature range from −67 °C to 204 °C, respectively. The lead wires of all the thermocouples were connected to a data logger. The temperatures were recorded at a specified equal time interval of 1 min while each experiment lasted for 60 min. First the refrigeration system of the chilling duct was run until a constant temperature of 1 °C was achieved. Then the test container was suspended in the test section of the air duct such that the conducting surfaces (copper covers) were parallel to the direction of flow of chilled air stream. The data logger was used to collect the transient temperature–time data. We note that the thermocouples were calibrated with accuracy of 0.01 °C in a test container before inserting in the flesh samples.

4. Computational procedure

The model Eqs. (1)–(9) were solved for predicting temperature time variations during cooling of solids with pure convection heat transfer from the surface of the solids. The co-ordinates system for the fish package is shown in Fig. 3. In order to establish finite difference scheme parameters which are accurate, reliable and efficient for heat transfer analyses during precooling of infinite slab-shaped bodies, a sample set of temperature–time data was chosen from the experimental

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat capacity</td>
<td>$C_p$</td>
<td>kJ/kgK</td>
<td>3.75</td>
</tr>
<tr>
<td>Surface heat transfer coefficient</td>
<td>$h$</td>
<td>W/m²K</td>
<td>79.0</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$k$</td>
<td>W/mK</td>
<td>0.530</td>
</tr>
<tr>
<td>Dry bulb temperature</td>
<td>$T_{db}$</td>
<td>°C</td>
<td>1</td>
</tr>
<tr>
<td>Initial temperature</td>
<td>$T_{in}$</td>
<td>°C</td>
<td>25</td>
</tr>
<tr>
<td>Half slab thickness</td>
<td>$X_0$</td>
<td>m</td>
<td>0.0127</td>
</tr>
<tr>
<td>Mass density</td>
<td>$\rho$</td>
<td>kg/m³</td>
<td>1052</td>
</tr>
<tr>
<td>Relative humidity of air</td>
<td>$\phi$</td>
<td>%</td>
<td>90</td>
</tr>
</tbody>
</table>
work with the relevant calculated thermophysical properties of Pangasius Sutchi fish as listed in Table 1. Calculations have been done for air-cooling with only heat transfer boundary conditions.

The experimental results for this slab-shaped sample during its air blast cooling considering only heat transfer (such as in canning and tinning) were compared for all the possible implicit finite difference schemes. The concept of least mean root square method of the error has been used as follows:

$$E(X) = \frac{1}{60} \sqrt{\sum_{j=1}^{60} (T_E(j) - T_p(j))^2}$$

Here, $X$ takes the values of 0, 0.2, 0.4, 0.6 and 0.8 for the locations of the sensors installed within the tested samples. $j$ refers to the temperature measurement number as well as to the predicted temperature number. Therefore it takes the values of 1, 2, ... up to 60. $E(0)$ is the error value at the center of the sample, when $E(0)=0$ it means the measured values from $j=1$ (first minute) until $j=60$ (last minute) exactly coincide with the predicted values. It is notable to mention here that the average error refers to the following equation:

$$E(\text{average}) = \frac{E(0) + E(0.2) + E(0.4) + E(0.6)}{4}.$$  

5. Results and discussion

First, Eq. (5) was examined by substituting $\beta=0, 0.2, 0.4, 0.6, 0.8$ and 1.0. A computer program has been developed in the visual Fortran for the system of Eqs. (1) (2) (3) (4) (5) (6) (7) (8) (9) and (13) to predict the temperature distributions versus time in the five locations inside the sample ($X=0, 0.2, 0.4, 0.6$ and 0.8) so as to compare the computed values with the experimental results as shown in Figs. 4 and 5. For the stability consideration ($((\Delta X)^2/\Delta x)$ has been selected as 6. The program was repeated many times to establish
a finite difference scheme, which is accurate, reliable and efficient for heat transfer analyses during precooling of infinite slabs. Calculations were repeated for different values of weighting factor ($\beta$) from 0 to 1 in step of 0.2 and the error values were obtained as shown in Fig. 6. Fig. 7 exhibits the variation of $n$ with and accuracy at constant $(\Delta X)^2/\Delta \tau$ for fully explicit, fully implicit and Crank Nicholson schemes. The minimal error could be obtained with $n = 10$ for all the used schemes. All the schemes give the same value of error in the range of $0 \leq n \leq 45$, beyond this range the implicit scheme will be the most accurate scheme then the explicit secondly whereas Crank Nicholson’s scheme is the least accurate scheme.

Furthermore, Fig. 6 shows the optimum value of $\beta$ with constant $n$ that yields, minimum processing time and high accuracy on the sensor locations starting from the center of the slab, $E(0)$, toward the surface of the sample with 4 equidistant steps. The computational results revealed that the processing time for implicit scheme is 36 times that of the explicit scheme. The error value at any sensor location decreases very slightly with $n$, and the minimal one is obtained from the sensor fixed at the center whereas the error increases with the sensor distance from the center of the infinite slab. It is notable to mention that the average error coincides very well with that incorporated with the sensor fixed at the first step from the slab center.

6. Conclusions

As presented above, the present work used the explicit finite difference scheme, implicit finite difference scheme and implicit explicit finite difference schemes with varying weighing factors. We can summarize some key findings as follows:

- The simple explicit finite difference scheme with $n = 10$ and $(\Delta X)^2/\Delta \tau = 6$ gives reliable and accurate results for making thorough heat analyses during air blast precooling of infinite slab-shaped packages of freshwater Pangasius Sutchi fish.
• As \( n \) increases from 5 to 10, the accuracy increases. And the value of accuracy starts decreasing due to truncation error.
• The processing time multiples 36 times when choosing implicit rather than explicit with insignificant improvement in accuracy.
• The computed temperature in this scheme is as accurate as nearer to the center of the sample.
• The proposed scheme is valid for the space coordinate \( 0 \leq X \leq 0.6 \) and \( 0.2 \leq \tau \leq 2.99 \).

Nomenclature

- \( Bi \): Biot number \( (hx_o/k_{fish}) \)
- \( E \): error value at any sensor location
- \( h \): surface film conductance \( (W/m^2 K) \)
- \( k \): thermal conductivity of product \( (W/mK) \)
- \( Nu \): Nusselt number \( (hx_o/k_{air}) \)
- \( Pr \): Prandtl number \( (\mu c_p/k_{air}) \)
- \( Re \): Reynolds number \( (\rho Vx_o/\mu) \)
- \( T \): temperature \( (^\circ C) \)
- \( t \): time \( (s \text{ or } \text{min}) \)
- \( X \): dimensionless coordinate \( (x/X_0) \)
- \( x \): distance from center \( (m) \)
- \( X_0 \): half thickness of infinite slab

Greek letters

- \( \theta \): dimensionless temperature \( [(T-T_{db})/(T_{in}-T_{db})]\)
- \( \tau \): Fourier number \( (\alpha t/X_0^2) \)
- \( \alpha \): thermal diffusivity of product \( (m^2/s) \)
- \( \beta \): weighting factor

Subscripts and superscripts

- \( db \): dry bulb temperature of cooling medium
- \( E \): experimental
- \( i \): space step
- \( in \): initial
- \( j \): time step
- \( o \): zero time
- \( P \): computed

References


