Numerical study of axial heat conduction effects on the local Nusselt number at the entrance and ending regions of a circular microchannel

Mohammad Rahimi, Reza Mehryar*

Department of Mechanical Engineering, Shiraz University of Technology, Modarres Blvd., Shiraz, Iran

Abstract

The effects of duct wall thermal conductivity and thickness on the local Nusselt number at the entrance and ending regions of a circular cross-section microchannel in a conjugate heat transfer problem are numerically investigated. The simultaneously developing laminar flow with a constant heat flow rate per unit length on the outside surface of the duct wall is considered. The results show that the axial heat conduction in the duct wall decreases the local Nusselt number at the entrance and ending regions of the microchannel with respect to their values for microchannels without conjugate effects. The effects of various parameters on the local Nusselt number at the ending region of the microchannel are analyzed and three generalized correlations are proposed for predicting the ending length and corresponding local and average Nusselt numbers. Comparisons with other numerical results show suitable accuracy of the correlations.

1. Introduction

Heat transfer in macrochannels can suitably be described by standard theories and correlations, but scaling effects in microchannels such as entrance effects, conjugate heat transfer, viscous heating, electric double layer (EDL) effects, temperature dependent properties, surface roughness, rarefaction and compressibility effects, often negligible in macrochannels, may now have a significant influence and have to be accounted for [1].

Conjugate heat transfer is one of the most important issues in heat transfer and fluid flow in micro-scale problems. In forced convection heat transfer problems in macroscale channels, the effect of conjugate heat transfer in channel walls can be neglected because the wall thickness is usually very small compared to the hydraulic diameter [2], but in micro-scale channels, wall thickness is significant compared to the hydraulic diameter and effect of axial heat conduction in the channel walls should be considered.

Conduction heat transfer in the walls of microchannels can get a quite multidimensional character. The wall heat flux density, for small Reynolds numbers, becomes strongly non-uniform with more amount of heat transfer to the fluid flow at the entrance of the microchannel [3] even for constant heat flux boundary conditions. The comparison between experimental and numerical results in various scientific researches shows that if in calculation of convective heat transfer coefficient from experimental data the effects of conjugate heat transfer phenomenon are neglected, the corresponding Nusselt numbers will generally be underestimated [2]. Moreover, the axial wall heat conduction causes the discrepancies between microchannel heat transfer results and predictions of standard correlations valid for conventional size channels [4,5]. It should be noted that the standard correlations for heat transfer in channels are valid under the idealized boundary conditions such as constant wall temperature or constant wall heat flux and moreover they are usually obtained for fully developed flows. But in micro-scale channels, it is difficult to achieve such conditions because wall conduction has a strong influence on the heat patterns particularly at low Reynolds number and should be accounted for [1].

In recent years, a lot of researches have been done numerically and experimentally in order to estimate the effects of axial heat conduction in the duct. For example, Mori et al. [6,7] studied the effects of axial wall heat transfer on the convective heat transfer between parallel plates and in circular cross-section microchannels under the boundary conditions of the first and the second kind, but they ignored this effect at the ending region. In another work, Faghri and Sparrow [8] investigated the simultaneous wall and fluid axial conduction in laminar pipe-flow heat transfer and
proposed a criterion for considering the importance of the axial heat conduction in the duct wall. In a similar research, Zarifcheh et al. [9] studied numerically the both effects of wall and fluid axial conduction on the laminar heat transfer in the circular cross-section channels. Barozzi and Pagliarini [10] investigated analytically the fluid flow in thick-walled ducts with two-dimensional conduction. Maranzana et al. [3] defined a dimensionless number, M, which was the ratio of the axial heat conduction in the solid walls of the duct to the heat convection in the fluid. They concluded that for $M < 0.01$, the axial heat conduction in the solid walls can be neglected. Gamart et al. [11] investigated wall conduction and heat conduction in rectangular microchannels both experimentally and numerically with a three-dimensional model. They found a significant reduction in Nusselt number when the hydraulic diameter was less than 1 mm. Celata et al. [12] analyzed experimentally and numerically the heat transfer parameters in circular microchannel with inside diameter ranging from 50 $\mu$m to 528 $\mu$m. They found some decrease in Nusselt number with respect to the analytical value, $Nu = 4.364$, specially for low Reynolds number flows. They attributed this reduction to a heat loss term. Nonino et al. [2] investigated conjugate forced convection and heat conduction in the circular microchannels with a finite element procedure. They modeled a simultaneously developing laminar flow of a constant property fluid in microchannels of different lengths. They analyzed effects of wall thermal conductivity, wall thickness, entrance length and heat loss through the end section of a microchannel. Nonino et al. [2] studied numerically the effects of wall axial heat conduction in a conjugate heat transfer problem. They modeled a simultaneously developing laminar flow and heat transfer in straight thick wall of the circular tube with constant outside wall temperature.

Several parameters in the conjugate heat transfer such as wall thickness and wall thermal conductivity have significant effects on the convective heat transfer performance in the microchannels. But a few researches have quantitatively investigated the effects of axial wall conduction on the local Nusselt number specially at the ending region of the duct.

In the present work, the effects of wall axial heat conduction on the local Nusselt number for low Reynolds numbers with simultaneously developing laminar flow and heat transfer are numerically investigated. A constant heat flow rate per unit length is applied on the outside surface of the duct wall. A self-coded CFD (computational fluid dynamic) program has been used for solving the Navier–Stokes and energy equations.

2. Theory and problem description

An axi-symmetric circular cross-section microchannel of length $L$ is considered in this research as shown in Fig. 1. Fluid enters the duct with uniform velocity $U_{in}$, and temperature, $T_{in}$. Duct outside to inside diameter ratio $D_o/D_s$ and solid wall to fluid thermal conductivity ratio, $k_s/k_f$ will be changed and their effects on the local Nusselt number will be investigated. For all cases, constant heat flow rate per unit length is considered on the outside surface of the duct wall.

In micro-scale flow, different flow regime may be happened depending on Knudsen number, $Kn = \frac{\lambda}{D_o}$. This number is usually used to check whether the fluid can be considered as continuum and, hence, if the Navier–Stokes equations can still apply. Gad-el-Hak [14] proposed the following criteria:

- $Kn<10^{-3}$: no-slip flow;
- $10^{-3} < Kn<10^{-1}$: slip flow;
- $10^{-1} < Kn<10$: transition flow;
- $Kn>10$: free-molecule flow.

In this research, no slip flow condition is adopted because the Knudsen numbers in the simulations are less than $10^{-3}$. Fluid properties variation can also be neglected, which is a reasonable assumption when the Reynolds number is equal to or larger than 50 [15].

Viscous heating can influence heat transfer in microchannels, depending on the flow and boundary conditions, and should always be checked. With considering the Morini criteria [16], viscous heating is usually important when Reynolds number is greater than 1000. So in the present simulations, the effect of viscous heating is also neglected. So in the present work, the two dimensional axi-symmetric steady state Navier–Stokes and energy equations were solved with following assumptions to describe the flow and heat transfer in whole regions.

- Laminar flow.
- Incompressible fluid flow and constant thermal properties of the solid and fluid.
- Negligible effects of body and electromagnetic forces.
- No slip flow and no temperature jump.

The governing equations that are simplified and non-dimensionalized for these conditions, are continuity equation (1), momentum equations (2) and (3) in longitudinal and radial directions and energy equations (4) and (5) in the fluid and solid regions respectively.

$$\frac{\partial U}{\partial X} + \frac{1}{R} \frac{\partial (RV)}{\partial R} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial R} = \frac{\partial P}{\partial X} - \frac{2}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) \right) \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial R} = \frac{\partial P}{\partial X} - \frac{2}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial V}{\partial R} \right) - \frac{V}{R} \right) \quad (3)$$

$$U \frac{\partial T_s}{\partial X} + V \frac{\partial T_s}{\partial R} = \frac{2}{Pr} \left( \frac{\partial^2 T_s}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial T_s}{\partial R} \right) \right) \quad (4)$$

$$\frac{\partial^2 T_s}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial T_s}{\partial R} \right) = 0 \quad (5)$$

Where $X = x/r_s$, $R = r/r_s$, $U = u/u_{in}$, $V = v/u_{in}$, $P = p/p_{in}$, $T = k_t(T - T_{in})/q_{sw}$ and $q = q_{sw}/q_{in}$.

These equations are solved with the following boundary conditions:

![Fig. 1. Schematic view of computational domain and boundary conditions.](image-url)
results of Nonino et al. [2] for two different cases in Fig. 3. Fig. 3a is for the case $D_o/D_i = 1$, $Re = 50$, $Pr = 5$ and $L/D_i = 25$ and Fig. 3b is for the case $D_o/D_i = 3$, $k_s/k_f = 250$, $Re = 200$, $Pr = 5$ and $L/D_i = 25$. These figures show good accuracy of numerical method.

4. Results and discussion

Different parameters such as Reynolds number, Prandtl number and wall boundary condition can influence local Nusselt number at the entrance region of a duct. However when the dimensions of the duct decrease to micro-scale, wall axial heat conduction is added to the previously mentioned parameters which has an important effect on the heat transfer patterns. The effect of the axial heat conduction is more relevant for short ducts, thick walls and low Reynolds numbers [2]. So, in the following sections, effects of different parameters on the wall axial heat conduction are investigated. For this purpose, the reference values for numerical simulations correspond to Table 2 unless another value is reported.

4.1. Entrance local Nusselt number

In this section, the effects of solid wall to fluid thermal conductivity ratio and outside to inside diameter ratio on the local Nusselt number at the entrance region of the microchannels are investigated. Fig. 4 shows the effect of the ratio $k_s/k_f$ on the local Nusselt number, inside wall heat transfer per unit length and temperatures along the microchannel. As it is observed in Fig. 4a, the local Nusselt number for “no wall” case where no axial heat conduction appears, has the highest value and it decreases with increasing solid wall to fluid thermal conductivity ratio. It should be noted that Nusselt number is proportional to the heat transfer coefficient which is defined as the ratio of heat flux to the fluid thermal conductivity ratio. It should be noted that Nusselt number is proportional to the heat transfer coefficient which is defined as:

\[ Nu = h_x D_i / k_f \]

Where

\[ h_x = \frac{q_{w}}{\pi D_i (T_w - T_b)} \]

In addition, the Nusselt number for a circular cross-section micro-channel is calculated as:

\[ Nu = \frac{h_x D_i}{k_f} \]

Non-dimensionalized form:

\[ q^* = \frac{q_{w}}{q_{w,ref}} = \frac{Re Pr \left( \frac{dR}{dx} \right)}{2 \frac{dt}{dx}} \]

Table 1

<table>
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<tr>
<th>Radial direction subdivisions</th>
<th>Fully developed Nusselt number</th>
<th>Simulation error (%)</th>
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<td>0.710</td>
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<td>25</td>
<td>4.368</td>
<td>0.090</td>
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</table>
temperature difference according to equation (8). So in order to express the reason of Nusselt number reduction with increasing the ratio \( k_i/k_f \), the variation of inside wall heat flux and temperature difference between inside wall surface and fluid bulk along the tube should be examined.

With increasing the ratio \( k_i/k_f \), axial heat conduction in the microchannel wall increases due to wall thermal resistance reduction. This effect causes more heat conduction toward the entrance region where temperature of the duct wall is lower due to higher convective heat transfer coefficient. Consequently at the entrance region, more heat enters the fluid as shown in Fig. 4b and also the inside wall temperature increases in comparison with “no wall” case as presented in Fig. 4c. In addition, the high heat flux at the entrance region causes high slope in the bulk temperature curve of the fluid according to equation (10) as shown in Fig. 4d, but increase of fluid bulk temperature is not noticeable in comparison to increase of inside wall temperature. So for the cases of high solid wall to fluid thermal conductivity ratio, the difference between inside wall surface temperature and fluid bulk temperature is increased in comparison with “no wall” case. Finally the Nusselt number is calculated by computing heat transfer coefficient from equation (8) and its value is reduced with increasing the ratio \( k_i/k_f \).

Besides the solid wall to fluid thermal conductivity ratio, the wall thickness has also an important effect on the local Nusselt number as it is shown in Fig. 5. It should be noted that as the axial heat conduction in the wall is proportional to the wall thermal conductivity and normal surface area, so the wall thickness has more or less the same effect as wall thermal conductivity on the local heat transfer coefficient. So increase of the ratio \( D_o/D_i \) increases the inside wall heat flux at the entrance region and the Nusselt number is reduced as shown in Fig. 5.

### 4.2. Ending region local Nusselt number

Generally, a duct is divided into two portions, the entrance and fully developed regions, but this is not valid for a micro-scale duct where axial heat conduction is not negligible. For a microchannel, axial heat conduction in the duct wall has significant effects on the heat transfer pattern at the ending region in comparison to macrochannels. So besides the entrance and fully developed regions there is an ending region where local Nusselt number considerably deviates from fully developed value.

The ending length and the deviation from fully developed Nusselt number increase with increasing the axial heat conduction in the wall. The reason of this phenomenon can be explained by Fig. 6. In this figure, an axi-symmetric microchannel with the effect of wall axial heat conduction has been schematically shown where a constant heat flow rate per unit length is imposed on the outside surface of the duct wall. The high heat flux at the entrance causes a reduction in heat flux at the ending region because the total heat on the inside and outside wall should be the same.

Fig. 7 shows the quantitative effect of solid wall to fluid thermal conductivity ratio on the heat transfer parameters at the ending region for conditions of Table 2. As it is observed in Fig. 7a, the local Nusselt number decreases about 10 percent at the ending region of the microchannel. When there is no axial heat transfer in the duct wall, for “no wall” case of this figure, the local Nusselt number is unchanged at the ending region in contrast to a thick wall case. Because of high local heat transfer coefficient and lower temperature of the wall at the entrance, high amount of heat is transferred in the axial direction opposed to the fluid flow direction as explained in previous section. The inside surface heat flux in the middle portion of the microchannel is uniform and is proportional to the outside surface heat flux with a coefficient equals to the outside to inside surface area ratio. Therefore the Nusselt number in this region is equal to the standard value, 4.364. At the ending region of the duct, the heat transfer per unit length on the inside wall surface decreases as shown in Fig. 7b because the total surface heat transfer at the inside wall should be equal to the total heat imposed on the outside surface, and so the heat transfer coefficient is deviated from the fully developed value. The local Nusselt number decreases proportionally to the inside wall heat flux, because the inside wall temperature and fluid bulk temperature do not change considerably at the ending region as shown in Fig. 7c and d. The ending length and the Nusselt number reduction value depend on the amount of axial heat conduction through the wall which is a function of microchannel geometry, flow conditions and thermal conductivity of wall. It should be noted that, although heat flux deviation at the ending region is less than its deviation at the entrance, the ending length is longer than the entrance length. So the effect of ending length must be considered on the prediction of average Nusselt number and total absorbed heat in fluid.

### Table 2

Reference values for present numerical simulations.

<table>
<thead>
<tr>
<th>( L/D_i )</th>
<th>( D_o/D_i )</th>
<th>( k_i/k_f )</th>
<th>( Re )</th>
<th>( Pr )</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>2.50</td>
<td>337.33</td>
<td>99.50</td>
<td>6.99</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison of the local Nusselt number with another numerical results of Nonino et al. [2] for: (a) \( D_o/D_i = 1 \), \( Re = 50 \), \( Pr = 5 \) and \( L/D_i = 25 \) and (b) \( D_o/D_i = 3 \), \( k_i/k_f = 25 \), \( Re = 200 \), \( Pr = 5 \) and \( L/D_i = 25 \).
4.3. The ending length

Various correlations have been proposed in scientific papers in order to calculate Nusselt number at the entrance and fully developed regions of the microchannels [21]. These correlations are usually used to estimate the value of average Nusselt number in the duct for some applications such as microchannel heat sinks selection and design. But it is not precise enough, because of Nusselt number deviation at the ending region. In order to obtain a precise average Nusselt number for a microchannel, it should be calculated considering Nusselt number and corresponding length at the entrance, fully developed and ending regions. In the literature, these values of ending region have not been yet estimated.

So in this research, the length of ending region is estimated by defining it as the distance between tube end and the first point downstream the fully developed region where the local Nusselt number is 99.8 percent of fully developed value. This ending length depends on the flow conditions, the tube geometry and thermal conductivity of the wall. So dimensionless form of ending length \( L^*_e \), may be defined as:

\[
L^*_e = \frac{L_e}{Re Pr D_i}
\]  

(11)

It can be expected that the dimensionless ending length will be a function of parameters related to flow and wall axial heat conduction. A parameter was previously introduced by Maranzana et al. [3] and named axial conduction number, M, which was used in several papers [2, 22, 23].

Fig. 4. Effect of the ratio \( k_s/k_f \) for the case \( L/D_i = 300, D_o/D_i = 2.5, Re = 99.5 \) and \( Pr = 6.99 \) on the: (a) Local Nusselt number; (b) dimensionless inside wall heat transfer per unit length; (c) dimensionless inside wall temperature; (d) dimensionless fluid bulk temperature.

Fig. 5. Effect of the ratio \( D_o/D_i \) on the local Nusselt number for the case \( L/D_i = 300, k_s/k_f = 337.33, Re = 99.5 \) and \( Pr = 6.99 \).

Fig. 6. A schematic view of axial heat conduction effect on the inside wall heat transfer per unit length.
\[
M = \left(\frac{k_w}{k_f}\right) \left(\frac{D_o^2 - D_i^2}{D_iL}\right) \frac{1}{Re \cdot Pr} \tag{12}
\]

In order to find a correlation for the dimensionless ending length, \(L_e^*\), its variation is shown in Fig. 8 versus \(M\) for different cases listed in Table 3. As it is observed, \(M\) is not a suitable variable because the curve is not monotonic. It should be noted that the ending length is not dependent on the tube length physically. So a new dimensionless number, \(M^*\), which is independent of tube length, is defined as equation (13) after several trial and errors.

\[
M^* = M \frac{L}{D_iRePr} = \left(\frac{k_w}{k_f}\right) \left(\frac{D_o^2 - D_i^2}{D_i^2}\right) \frac{1}{Re^2 \cdot Pr^2} \tag{13}
\]

It should be noted that Maranzana number is the ratio of heat conduction in the wall to the advection in the fluid, as it is reported [11]. But the meaning of the modified axial heat conduction number, \(M^*\), is a little different from Maranzana number. \(M^*\) is product of two different effects, the ratio of wall to fluid heat conduction and the reciprocal of Peclet number to the second power. If wall thermal conductivity or its thickness is high, the wall to fluid heat conduction ratio is high and so the value of \(M^*\) is relatively large. On the other hand, the presence of Peclet number with the second power expresses a higher effect of this term of equation (13) on the importance of conjugate heat transfer in microchannels.

With the definition of this number, the dispersed points of Fig. 8 are rearranged in a monotonic form as shown in Fig. 9. This curve confirms the precise definition of equation (13) and can be fitted by correlation (14).

\[
L_e^* = \frac{0.1}{0.02984M^{0.64} - 0.0064} \tag{14}
\]

This correlation has been tested for different Reynolds number in laminar regime, but it should be noticed that for high Reynolds number flow in laminar regime, the required entrance length is too long with respect to ordinary size of microchannels. In addition, the effect of conjugate heat transfer in microchannels is negligible at high Reynolds number [16]. So in this research, as shown in Table 3, the range of Reynolds numbers has been considered between 50 and 500 and the range of Prandtl numbers is about 2–35 which is an applicable range for the liquids that are usually used in microchannels such as water and Ethylene Glycol.
In order to evaluate the accuracy of correlation (14), the ending length obtained by this correlation was compared to some other direct numerical simulations. The results showed a good accuracy with a maximum error of 2.61 percent.

### 4.4. Nusselt number correlations

Considering the simulation results of all seventeen cases of Table 3, a generalized correlation, equation (15), for the local Nusselt number at the ending region of the microchannel was obtained as a function of dimensionless distance from end of the duct, $x_e^*$, and the previously defined dimensionless number, $M'$:

$$ Nu_x = a \exp \left( b x_e^* \right) - c \exp \left( -d x_e^* \right) $$

(15)

Where

$$ a = 4.364 $$

(16)

$$ b = 1.736 \times 10^{-4} M'^{-0.7546} - 7.84 \times 10^{-4} $$

(17)

$$ c = 4.086 \times 10^{-2} M'^{-0.3014} + 0.6501 $$

(18)

This correlation is applicable when the duct length is equal to or greater than sum of entrance and ending lengths of the duct. In addition, the Knudsen number should be lower than $10^{-3}$ corresponds to no slip condition.

In order to ensure the accuracy of correlation (15), the corresponding Nusselt number was compared to the results of Nonino et al. [2] for two cases ($D_o/D_i = 2$ and $3$ where $k_s/k_f = 250$, $Re = 50$, $Pr = 5$, $L/D_i = 100$) as it is shown in Fig. 10. These comparisons show good accuracy of correlation (15) with other numerical results.

Integrating the local Nusselt number correlation, an average Nusselt number for the microchannel ending region can be obtained:

$$ Nu_{ave} = \frac{d}{L_e^*} \left[ a \exp \left( b L_e^* \right) - c \exp \left( -d L_e^* \right) \right] $$

(20)

Where $a$, $b$, $c$, and $d$ are same as equations (16)–(19) and $L_e^*$ is ending length of the duct, calculated from equation (14).

### 5. Conclusion

The effects of axial heat conduction on the local Nusselt number in circular cross-section microchannels with simultaneously

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**Table 3**

Microchannels dimensions and the related conditions of simulation cases.

<table>
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<th>Case</th>
<th>$k_s/k_f$</th>
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<th>$Re$</th>
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**Fig. 9.** Variation of the dimensionless ending length with presently defined dimensionless number, $M'$.

**Fig. 10.** Comparison between correlation (15) and results of Nonino et al. for $k_s/k_f = 250$, $Re = 50$, $Pr = 5$, $L/D_i = 100$ and (a) $D_o/D_i = 2$ (b) $D_o/D_i = 3$. 

$$ d = 3.559 M'^{-0.5565} - 0.3743 $$

(19)
developing flow and a constant heat flow rate per unit length on the outside surface of the wall have been numerically investigated. The results showed that the axial heat conduction in the duct wall due to wall thermal conductivity and its thickness causes a reduction in the local Nusselt number at the entrance region and also a deviation in the local Nusselt number at the ending region of the microchannel. By defining a new dimensionless number, three correlations for ending length, local and average Nusselt number were represented. These correlations were taken into account the effects of solid wall to fluid thermal conductivity ratio, outside to inside duct diameter ratio, and Reynolds and Prandtl numbers. The results show a good accuracy of these correlations with other numerical results. In addition, a new dimensionless axial conduction number was presented in this paper. It is proportional to the solid wall to fluid axial conduction ratio and the reciprocal of Peclet number to the second power.

References


Glossary

c_p: fluid specific heat, J/(kg·K)
D: duct diameter, m
h: convective heat transfer coefficient, W/(m²·K)
Kn: Knudsen number, —
d: duct diameter, m
k: thermal conductivity, W/(m·K)
L: total length of duct, m
L_e: ending length of duct, m
L_d: dimensionless ending length of duct
M: previously defined axial conduction number
M_e: presently defined axial conduction number
m: mass flow rate, kg/s
Nu: Nusselt number
p: deviation from hydrostatic pressure, Pa
Pe: dimensionless hydrostatic pressure, — p/ρu²_m
Pr: Peclet number, — u/ν
q: heat transfer per unit length, W/m
q_e: dimensionless inside wall heat transfer per unit length, — q_e/q_m
x: cylindrical coordinate, m
R,X: dimensionless coordinate
Re: Reynolds number, — ρu_dD_d/μ
T: temperature, K
T_e: dimensionless temperature, — k_e(T – T_in)/q_m
u: fluid velocity in axial direction, m/s
V: dimensionless fluid velocity in axial direction
x: distance from outlet
X_e: dimensionless distance from inlet, — x/(Re Pr D)
x_e: dimensionless distance from outlet, — x_e/(Re Pr D)

Greek symbols
µ: mean free path of fluid molecules, m
µ: dynamic viscosity, kg/(ms)
ρ: density, kg/m³

Subscripts and superscripts
ave: average
b: fluid bulk
f: fluid
h: hydraulic
i: inside
in: inlet
o: outside
out: outlet
s: solid
w: wall
x: local